

Information Economics

Suggested Solution for Problem Set 1

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1. (a) $\nabla f(x_1, x_2) = \begin{bmatrix} 10x_1^4 + 6x_1x_2 + 3 \\ 3x_1^2 - 3x_2^3 \end{bmatrix}$, $\nabla^2 f(x_1, x_2) = \begin{bmatrix} 40x_1^3 + 6x_2 & 6x_1 \\ 6x_1 & -6x_2^2 \end{bmatrix}$.
- (b) $\frac{d}{dx}f(x) = \frac{3x^2 + 2}{x^3 + 2x}e^{3x} + 3\ln(x^3 + 2x)e^{3x}$.
- (c) $\int f(x)dx_2 = \frac{1}{3}x_1x_2^3 + \frac{1}{2}x_1e^{2x_2}$.
- (d) $\frac{d}{dx} \int_0^x (t^3 + 3x - 2)dt = x^3 + 6x - 2$.
- (e) $\mathbb{E}[X] = 3.2$, $\mathbb{V}[X] = \mathbb{E}[X^2] - \mathbb{E}[X]^2 = 3.36$.
- (f) Because we need $\int_0^2 kx^{1.5}dx = 1$, we have $k = \frac{5}{8\sqrt{2}}$. It then follows that $\mathbb{E}[X] = \int_0^2 xf(x)dx = \int_0^2 x^{2.5} \frac{5}{8\sqrt{2}} dx = \frac{10}{7}$.
- (g) Because $\frac{d^2}{dx^2}f(x) = \frac{15}{4}x^{\frac{1}{2}} + 6 > 0$ over $[0, \infty)$, it is convex over the region.
- (h) Because $\frac{d^2}{dx^2}g(x) = -x^{-2} + 4 \geq 0$ if and only if $x \in (-\infty, -\frac{1}{2}] \cup [\frac{1}{2}, \infty)$, it is convex over there.
2. (a) As shown in Figure 1, the area in gray is the feasible region. Obviously, it is not a convex set since there exists some points between point A and B that do not belong to the feasible region.

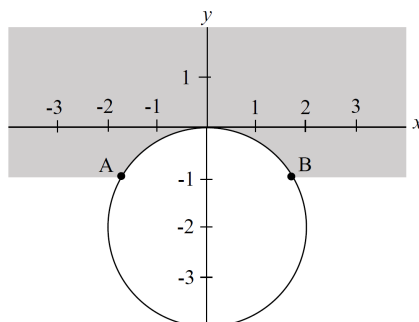


Figure 1: Graphical solution

- (b) This program is unbounded, so there is no optimal solution.
- (c) Yes, the point $(-\sqrt{3}, -1)$ is not a global maximum but a local one since there does not exist any point nearby that is greater than it.
- (d) No, there exists no point that is local maximum but not a global maximum.
3. Suppose that there are two points x and y , $x, y \in F \cap G$. Then we know $x, y \in F$ and $x, y \in G$. Since F and G are convex sets it follows that $z \in F$ and $z \in G$ where

$$z = \lambda x + (1 - \lambda)y, \lambda \in [0, 1].$$

Hence $z \in F \cap G$.

4. (a) The problem can be formulated as

$$\begin{aligned} \max_p \quad & \pi(p) = (p - c)a \left(1 - \frac{p}{b}\right) \\ \text{s.t.} \quad & 0 \leq p \leq 0. \end{aligned}$$

Since $\pi''(p) = \frac{-2a}{b} < 0$ and $p \in [0, p_0]$, we maximize a concave function over a convex feasible region. Therefore, the problem is a convex program.

(b) The optimal solution is

$$p^* = \begin{cases} \frac{b+c}{2} & \text{if } \frac{b+c}{2} \leq p_0 \\ p_0 & \text{otherwise} \end{cases}.$$

(c) p^* is increasing in the highest possible valuation b , because the seller can charge more when consumers' willingness to pay is higher. p^* is increasing in the unit cost c , because the seller will try to cover the additional cost by increasing the retail price. p^* has nothing to do with the total number of consumer a , because the total number of consumer will only affect the total profit but not the retail price.

5. (a) The problem can be formulated as

$$\begin{aligned} \max_q \quad & \pi(q) = r\mathbb{E}[\min\{q, D\}] + d\mathbb{E}[\max\{q - x, 0\}] - cq \\ \text{s.t.} \quad & q \geq 0. \end{aligned}$$

(b) First, we may rewrite $\pi(q)$ into

$$\pi(q) = r \left\{ \int_0^q x f(x) dx + \int_q^\infty q f(x) dx \right\} + d \int_0^q (q - x) f(x) dx - cq.$$

We then have

$$\begin{aligned} \pi'(q) &= r \left\{ [qf(q)] + [-qf(q) + \int_q^\infty f(x) dx] \right\} + d \int_0^q f(x) dx - c \\ &= r(1 - F(q)) + dF(q) - c \\ &= r + (d - r)F(q) - c, \end{aligned}$$

which implies that the optimal quantity q^* satisfies

$$F(q^*) = \frac{r - c}{r - d}.$$

(c) $F(q^*)$ is increasing in r . If the unit revenue increases, the newsvendor will have an incentive to order more quantity. $F(q^*)$ is decreasing in c . If the unit cost increases, the newsvendor will choose to order less to avoid from overstocking. $F(q^*)$ is increasing in d . If the salvage value increases, the newsvendor will have an incentive to order more because the recycling site shares the risk with him.