1. Under the DD structure, the two retailers play a Bertrand game. Suppose now they play a Cournot game by choosing order quantities q_1 and q_2, respectively. Given their choices, the market-clearing price is $p = a - q_1 - q_2$. Retailer i then earns $(p - w_i)q_i$, where w_i is the wholesale price chosen by manufacturer i. All players act to maximize their own profits.

   (a) Solve the second stage Cournot game and derive the equilibrium retail prices as functions of the wholesale prices.

   (b) Solve the first stage wholesale price game and derive the equilibrium wholesale prices.

   (c) Compare the equilibrium outcomes of DD and II (which results in $q_i^* = \frac{a}{3}$). Is DD or II better for the manufacturers? May we conclude that the better one will be an equilibrium?

2. In lecture videos, we solved the static channel structure game

M1 \ M2	I	D
I	$\frac{1}{(2-\theta)^2}$, $\frac{1}{(2-\theta)^2}$	$\left[\frac{4+\theta-2\theta^2}{2(2-\theta)(2-\theta^2)}\right]^2$, $\frac{2+\theta}{4(2-\theta)(2-\theta^2)}$
D	$\frac{2+\theta}{4(2-\theta)(2-\theta^2)}$, $\left[\frac{4+\theta-2\theta^2}{2(2-\theta)(2-\theta^2)}\right]^2$	$\frac{(2+\theta)(2-\theta^2)}{(2-\theta)(4-\theta-2\theta^2)^2}$, $\frac{(2+\theta)(2-\theta^2)}{(2-\theta)(4-\theta-2\theta^2)^2}$

We showed that when $0.708 < \theta < 0.931$, this static game is actually a prisoners' dilemma: The two firms may be better off by choosing DD together, but II is the unique Nash equilibrium.

(a) Suppose that $0.708 < \theta < 0.931$. What if the game is played dynamically, i.e., manufacturer 1 first sets its channel structure and then manufacturer 2 makes its decision by observing manufacturer 1's decision? Does the prisoners' dilemma go away or remain there?

(b) Suppose that $0.931 < \theta$. What if the game is played dynamically, i.e., manufacturer 1 first sets its channel structure and then manufacturer 2 makes its decision by observing manufacturer 1's decision? Will there still be multiple equilibria?

3. Consider the pricing games under the ID structure. In the first stage, manufacturer 2 sets its wholesale price w_2; in the second stage, manufacturer 1 and retailer 2 set their retail prices p_1 and p_2, respectively.

   (a) Given w_2, solve the second stage game. Note that this is a Bertrand game with asymmetric procurement costs.

   (b) Solve manufacturer 2's problem in the first stage.

4. In lecture videos, we first formulated the manufacturers' problems

$$\max_{w_i}\ w_i\left[\frac{1}{2-\theta}-\frac{(2-\theta^2)w_i-\theta w_{3-i}}{(2+\theta)(2-\theta)}\right],\quad i=1,2, \tag{1}$$

and then applied the FOC to require an equilibrium (w_1^*, w_2^*) to satisfy

$$\frac{1}{2-\theta}-\frac{2(2-\theta^2)w_i^*-\theta w_{3-i}^*}{(2+\theta)(2-\theta)}=0,\quad i=1,2. \tag{2}$$

The unique solution to the above two equations is

$$w_1^* = w_2^* = \frac{2+\theta}{4-\theta-2\theta^2}. \tag{3}$$

(a) Use the two equations in (2) to form a two by two linear system and verify that the solution in (3) is correct.

(b) As this is a symmetric game, in equilibrium the two manufacturers will choose the same wholesale price. Therefore, let's set $w_1^* = w_2^* = w^*$ in (2) and solve

$$\frac{1}{2-\theta} - \frac{2(2-\theta^2)w^* - \theta w^*}{(2+\theta)(2-\theta)} = 0.$$

Do you get the solution in (3)? If yes, will this simplification always work? If no, why?

(c) Alternatively, let's set $w_1^* = w_2^* = w^*$ in (1) and solve

$$\max_{w} \; w\left[\frac{1}{2-\theta} - \frac{(2-\theta^2)w - \theta w}{(2+\theta)(2-\theta)}\right].$$

Do you get the solution in (3)? If yes, will this simplification always work? If no, why?

(d) Compare your answers in Parts (b) and (c). Why they are the same or different?

5. Will the main results obtained with the two-channel model extend to a model with more than two channels? In particular, will pure decentralization still be an equilibrium when competition is high? Will there be new insights when there are more than two channels?

6. Find some other real examples of "exclusive retailers" in practice, some as company stores and some as franchise stores. Do manufacturers tend to open franchise stores under intense competition?