1. Consider the model in the section *The first model*. Suppose that θ now is distributed uniformly in $[a, b]$, where $0 \leq a < b < \infty$.

   (a) Formulate the demand function as a function of p.

   (b) Formulate the profit function as a function of p.

   (c) Derive the optimal price and the associated profit.

   (d) Show how a, b, and c affect the optimal price.

2. Consider the model in the section *Exogenous product quality*. Suppose that θ now is distributed uniformly in $[a, b]$, where $0 \leq a < b < \infty$.

   (a) Formulate the demand function as a function of p.

   (b) Formulate the profit function as a function of p.

   (c) Derive the optimal price and the associated profit.

   (d) Show how a, b, c, and q affect the optimal price.

3. Consider the model in the section *Exogenous product quality*. Suppose that θ now follows a continuous distribution characterized by the PDF f and CDF F.

   (a) Formulate the demand function as a function of p.

   (b) Formulate the profit function as a function of p.

   (c) Derive an optimality condition for an optimal price.

   (d) Prove that the optimal price increases in c and q or give a condition under which this is true.

4. Consider the *binary* model in the section *Endogenous product quality*. Suppose that the product is an information good. While there is no unit production cost, there is an R&D cost $\frac{cq^2}{2}$ to reach the product quality level q.

   (a) Formulate the two optimization problems for selling to all consumers or only the high-end consumers.

   (b) Solve the two optimization problems.

   (c) Give a condition under which serving all customers is better than serving only the high segment.

5. Consider the model in the section *Exogenous product quality*. Suppose that now the product is a network good, and a consumer's utility function of buying the product now becomes

$$\theta q - p + tx,$$

where x is the number of consumers buying the product and t is the degree of network externality. Assume that $\theta \sim \mathrm{Uni}(0, 1)$.

(a) Formulate the demand function as a function of p.

(b) Formulate the profit function as a function of p.

(c) Derive the optimal price, if possible, or derive an optimality condition for an optimal price.

(d) Show how c, q, and t affect the optimal price.