

Information Economics, Spring 2018

Pre-lecture Problems 7

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Note 1. The deadline of submitting the pre-lecture problem is **9:30 am, May 4, 2018**. Please submit a hard copy of your work to the instructor in class. Alternatively, you may submit a hard copy into the instructor's mailbox on the first floor of Management Building 2 by **9:10 am** of the same day. Late submissions will not be accepted. Each student must submit her/his individual work. Submit **ONLY** the problem that counts for grades.

Note 2. Please make your answer as clear (i.e., easy to read) as possible. We reserve the right to take away points when the correctness cannot be easily determined (e.g., when the writing is messy and cannot be easily understood).

1. (0 points) Consider a salesperson who may privately exert sales effort a . The cost of exerting effort a is $\frac{1}{2}a^2$. The sales quantity is $x \in \{0, 1\}$, which follows a Bernoulli distribution

$$\Pr(x = 1|a) = a = 1 - \Pr(x = 0|a).$$

In other words, the higher effort exerted by the salesperson, the higher probability the good is sold. Let the production cost be 0 and retail price be 1. Let the salesperson be the one that earns the sales revenue. Suppose that the salesperson's objective is to maximize his expected profit, find his optimal effort level and the associated profit.

2. (0 points) Following from the previous problem, suppose that the salesperson is hired by a retailer. The retailer offers the salesperson a contract $\{\alpha, \beta\}$, where $\alpha \in \mathbb{R}$ is the fixed salary and $\beta \in [0, 1]$ is the sales bonus. In other words, the salesperson's earning is $\alpha + \beta x$. Suppose that both players' objectives are to maximize each of their own expected profit.
 - (a) Suppose that the salesperson has selected the contract (α, β) , find his optimal effort and the associated profit as functions of α and β .
 - (b) Find the retailer's optimal contract.
 - (c) Does the salesperson earn any information rent? Briefly explain why.

3. (10 points) Consider a salesperson who may privately observe the market condition θ . The sales quantity is $x \in \{0, 1\}$, which follows a Bernoulli distribution

$$\Pr(x = 1|\theta) = \theta = 1 - \Pr(x = 0|\theta).$$

In other words, the better market condition, the higher probability the good is sold. θ follows another Bernoulli distribution

$$\Pr(\theta = \theta_L) = \frac{1}{2} = 1 - \Pr(\theta = \theta_H),$$

where $0 < \theta_L < \theta_H < 1$. Let the production cost be 0 and retail price be 1. Suppose that the salesperson is hired by a retailer. For each type of salesperson, the retailer offers a contract $\{\alpha, \beta\}$, where $\alpha \in \mathbb{R}$ is the fixed salary and $\beta \in [0, 1]$ is the sales bonus. In other words, the salesperson's earning is $\alpha + \beta x$. Suppose that both players' objectives are to maximize each of their own expected profit.

- (a) (2 points) Explain why the retailer should offer a menu of two contracts. Limit your answer to at most 50 words.
- (b) (4 points) Formulate the retailer's contract design problem.

Hint. There should be two IR constraints and two IC constraints.
- (c) (4 points) Find the retailer's optimal menu of contracts.

Hint. May you use the first-order condition to find an optimal solution? Why or why not?