Information Economics

Fall 2013

Suggested Solution for Homework 2

A1. The expected profit given the order quantity q is

$$\pi(q) = r\,\mathbb{E}\min\{D, q\} - cq + s\,\mathbb{E}\max\{q-D, 0\}$$

$$= r\left\{\int_0^q x f(x)\,dx + \int_q^\infty q f(x)\,dx\right\} - cq + s\int_0^q (q-x) f(x)\,dx.$$

Let $\pi'(q^*) = r q^* f(q^*) + r\left[-q^* f(q^*) + 1 - F(q^*)\right] - c + s\,F(q^*) = 0$

$$\Rightarrow F(q^*) = \frac{r-c}{r-s} \quad \text{or} \quad 1 - F(q^*) = \frac{c-s}{r-s}.$$

A2. (a) Let $\pi_i(q_i)$ be the profit function of firm i by supplying q_i, we have $\pi_i(q_i) = [a - (q_1 + q_2 + q_3) - c]\,q_i$. Suppose (q_1^*, q_2^*, q_3^*) is a Nash equilibrium, it must satisfy

$$\pi_1'(q_1^*) = 0 \iff 2q_1^* + q_2^* + q_3^* = a - c,$$

$$\pi_2'(q_2^*) = 0 \iff q_1^* + 2q_2^* + q_3^* = a - c, \text{ and}$$

$$\pi_3'(q_3^*) = 0 \iff q_1^* + q_2^* + 2q_3^* = a - c.$$

Summing the three equations up, we get $q_1^* + q_2^* + q_3^* = \frac{3}{4}(a-c)$.

This then implies $q_1^* = q_2^* = q_3^* = \frac{1}{4}(a-c)$. This is a unique equilibrium.

(b) When there are two firms, the equilibrium supply quantity is $\frac{1}{3}(a-c)$.

The equilibrium supply quantity goes down when the number of firms increases from two to three. This is because the room for firms to supply more becomes smaller.

A3. (a) The unique Nash equilibrium is $x_1^* = x_2^* = \frac{1}{2}$. At this point, unilateral deviation simply decreases one's profit. For any other (x_1, x_2) such that $x_1 \neq x_2$, firm 1 can be strictly better off by moving to $\frac{x_1 + x_2}{2}$ and thus it is not a Nash equilibrium.

(b) They should together locate at the median (50th percentile) so that at each side there are one half of consumers.

A4. (a) The retailer solves $\max_r (r - w_2)(1 - r) \Rightarrow r^*(w_2) = \frac{1 + w_2}{2}$.

The wholesaler solves $\max_{w_2} (w_2 - w_1)\left(\frac{1 - w_2}{2}\right) \Rightarrow w_2^*(w_1) = \frac{1 + w_1}{2}$.

The manufacturer solves $\max_{w_1} (w_1)\left(\frac{1 - w_1}{4}\right) \Rightarrow w_1^* = \frac{1}{2}$.

This then implies that $w_2^* = w_2^*(w_1^*) = \frac{3}{4}$ and $r^* = r^*(w_2^*) = \frac{7}{8}$.

(b) Under decentralization, the equilibrium retail price is $\frac{3}{4}$ and the equilibrium sales quantity is $\frac{1}{4}$. Under integration, the equilibrium retail price is $\frac{1}{2}$ and the equilibrium sales quantity is $\frac{1}{2}$. As more consumers can buy this product at a lower price, integration benefits consumers.

A5. Let q^D be the system-optimal quantity under direct sales and q^I and w^I be the equilibrium quantity and wholesale price under indirect sales, we have $F(q^I) = 1 - \frac{w^I}{r} < 1 - \frac{c}{r} = F(q^D)$, where $w^I > c$ comes from the fact that the manufacturer is a profit maximizer and setting c as the wholesale price generates no profit. Then as $F(\cdot)$ is increasing, $q^I < q^D$.

B1. (a) $\pi_R(Q) = (p+s)\left\{ \int_0^Q x f(x)\,dx + \int_Q^\infty Q f(x)\,dx \right\} - wQ$

$\pi_R'(Q) = (p+s)\left(1 - F(Q)\right) - w$

$\pi_R''(Q) = -(p+s) f(Q) \leq 0 \quad \forall Q \geq 0 \quad \Rightarrow \quad \pi_R(Q)$ is concave.

(b) In this case, we have

$$\pi_R(Q) = p\int_0^T x f(x)\,dx + (p+s)\int_T^Q x f(x)\,dx + (p+s)\int_Q^\infty Q f(x)\,dx - wQ$$

$$\pi_R'(Q) = (p+s)\left[Q f(Q) - Q f(Q) + 1 - F(Q)\right] - w$$

$$= (p+s)\left(1 - F(Q)\right) - w$$

(c) If $Q < T$, $\pi_R(Q) = p\left\{ \int_0^Q x f(x)\,dx + \int_Q^\infty Q f(x)\,dx \right\} - wq$

and $\pi_R'(Q) = p(1 - F(Q)) - w$. Therefore, if Q_1^* and Q_2^* are the respective order quantities that satisfy $\pi_R'(Q) = 0$ when $Q < T$ and $Q \geq T$, we have $1 - F(Q_1^*) = \frac{w}{p}$ and $1 - F(Q_2^*) = \frac{w}{p+s}$.

To show that it is possible for $\pi_R(Q)$ to be nonconcave, consider the case with $p = s = 3$, $w = 2$, $T = \frac{1}{2}$, and $f(x) = 1$ for $x \in [0,1]$.

In this case, $Q_1^* = \frac{1}{3}$ and $Q_2^* = \frac{2}{3}$. It is then clear that $\pi_R(Q_1^*) > \pi_R(T)$, $\pi_R(Q_2^*) > \pi_R(T)$, and $Q_1^* < T < Q_2^*$, which imply $\pi_R(Q)$ is nonconcave.

B2. (a) $\pi_R(q,t) = r\left\{\int_0^q x f(x)\,dx + q(1-F(q))\right\} - t$ if the retailer accepts the contract. He should accept the contract if and only if $\pi_R(q,t) \geq 0$.

(b) $\max\limits_{q\geq 0,t} \; t - cq$

s.t. $\pi_R(q,t) \geq 0$.

(c) As long as $r\left\{\int_0^{q^*} x f(x)\,dx + q^*(1-F(q^*))\right\} - cq^* \geq 0$, where q^* satisfies $1-F(q^*) = \frac{c}{r}$, the system can generate a nonnegative profit by ordering the system-optimal quantity q^*. Then channel coordination can be achieved. For example, if the manufacturer offers q^* units with transfer $r\left\{\int_0^{q^*} x f(x)\,dx + q^*(1-F(q^*))\right\} = t^*$, then $\pi_R(q^*, t^*) = 0$ and the retailer will accept the contract. Arbitrary profit splitting can also be achieved by lowering t^*.

B3. Full returns with full credits will always induce a too high equilibrium inventory level. To see this, note that if $R=1$ and $c_2 = c_1$, the retailer will order Q_R^* such that $F(Q_R^*) = 1$ (from equation (7)). However, the channel-optimal quantity Q_T^* satisfies $F(Q_T^*) = \frac{p+g_2-c}{p+g_2-c_3} < 1$, which implies $Q_R^* > Q_T^*$.