

# Information Economics, Spring 2013

## Homework 2a

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**Note 1.** This homework is due **5:00 pm, September 27, 2013**. Please submit a hard copy into the instructor's mail box. As each team only needs to submit one copy, please indicate the names and student IDs of all team members on the first page.

**Note 2.** Each team must have **exactly three** students unless a special approval is obtained.

- (10 points) Consider a newsvendor problem with a random demand  $D$  whose cdf is  $F$ , unit production cost  $c$ , and unit retail price  $r$ . Besides of these, there is a positive salvage value  $s$  for each unit of unsold product. Find the formula that determines the optimal order quantity.
- (10 points) Consider a three-player Cournot competition, in which three firms simultaneously set their supply quantities  $q_1$ ,  $q_2$ , and  $q_3$ , the unit price is  $a - Q = a - (q_1 + q_2 + q_3)$  for some  $a > 0$ , and the unit production cost is  $c < a$  for all the three firms.
  - (5 points) Find the equilibrium supply quantities. Find all if there are more than one.
  - (5 points) When the number of firms goes from two to three, does the equilibrium supply of a single firm increase or decrease? Intuitively explain why.
  - (0 points) Convince yourself that when the number of firms goes to infinity, each firm will earn no profit in equilibrium.
- (10 points) Consider the following game in which two firms compete in a so-called Hotelling line. Two firms, 1 and 2, simultaneously set their store locations  $x_1$  and  $x_2$  within a line segment  $[0, 1]$ . Consumers spread on the line segment uniformly. Once the locations are set, a consumer will go to the store that is closer to her; if the two stores are equally close, she will go to either one with probability  $\frac{1}{2}$ . Each firm wants to maximize the expected number of consumers going into its store.
  - (5 points) Find the equilibrium locations. Find all if there are more than one.
  - (5 points) If consumers do not spread uniformly, what should the two firms do?
- (10 points) Recall the supply chain pricing problem discussed in Lecture 2.3. Still assume that  $A = B = 1$  and  $C = 0$ .
  - (5 points) Suppose there are three firms in the supply chain, a manufacturer, a wholesaler, and a retailer. The manufacturer first charges the wholesaler a unit price  $w_1$ , the wholesaler then charges the retailer a unit price  $w_2$ , and lastly the retailer charges consumers a unit retail price  $r$ . Find the equilibrium outcome.
  - (5 points; this part is not related to the first part) In the Cournot game, we find that a *horizontal integration* benefits the firms but harms consumers. How about a *vertical integration* that integrating the two firms in our supply chain pricing problem? Mathematically show it and intuitively explain why.
- (10 points) Consider the following *indirect newsvendor* problem: A manufacturer sells a product to a newsvendor retailer by setting a unit wholesale price  $w$ . The retailer, facing a random demand  $D$  and a fixed retail price  $r$ , then sets the order quantity  $q$ . Suppose there is a unit production cost  $c$ , the manufacturer can produce those products after the retailer places an order (i.e., in a MTO fashion), and each firm wants to maximize its expected profit. Without assuming  $D$  to follow any specific distribution, show that the equilibrium order quantity is smaller than that in a direct newsvendor setting.