Information Economics, Fall 2013
Homework 5

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Note 1. This homework is due ***8:30 am, November 25, 2013***. Please submit a hard copy into the instructor's mail box. As each team only needs to submit one copy, please indicate the names and student IDs of all team members on the first page.

Note 2. Each team must have ***exactly three*** students unless a special approval is obtained.

1. (10 points) Following what we did in class, rigorously prove Lemma 1 of Kung and Chen (2011), starting from formulation. You may certainly check the proof in the paper. However, as you may see that the proof therein is quite short and omits a lot of details, you should provide all the details step by step. The grades will be given according to how your proof shows your understanding.

2. (30 points; 10 points each) In the basic model of Kung and Chen (2011), a downstream player is willing to pay any amount of fixed payment to its upstream player. When this is not the case, i.e., an agent cannot or does not want to pay an upfront payment to its principal, we say that the agent is protected by *limited liability*. In this problem, we study a model with limited liability.

   (a) Consider a manufacturer-retailer relationship in which the manufacturer decides the wholesale price w and fixed payment t, the retailer decides the order quantity q, the unit production cost is c, the unit retail price is p, the random demand is $D \sim \text{Uni}(0,1)$. Suppose both firms are expected profit maximizer and the retailer will pay any amount of t as long as his expected profit is nonnegative. Find the equilibrium wholesale price, fixed payment, and order quantity.

   (b) Solve the problem by assuming that the manufacturer cannot charge the fixed payment. Compare your results with those in Part (a). Does limited liability increase or decrease the retailer's expected profit and system efficiency? Briefly provide economic intuitions.

   (c) Suppose the retailer is *partially* protected by limited liability, i.e., the manufacturer can charge a fixed payment $t \le L$, where $L \ge 0$ is the maximum amount the retailer may pay. For what value(s) of L the equilibrium is the same as that in Part (a)? For what value(s) of L the equilibrium is the same as that in Part (b)?

3. (45 points) Consider another model for the same problem studied in Kung and Chen (2011). There are still a manufacturer, a reseller, and a salesperson forming a supply chain. The salesperson still determines his effort level $a \ge 0$, which incurs a cost $\frac{1}{2}a^2$. However, the sales outcome $x \in \{0,1\}$ is now binary. The probability for selling a unit is $\Pr(x=1) = \theta a$, so a higher effort makes it more likely to sell a unit. The market condition $\theta \in \{\theta_L, \theta_H\}$ is also binary, where $0 < \theta_L < \theta_H < 1$. The salesperson observes θ and a, the knowledgeable reseller observes θ, the diligent reseller observes a, and the manufacturer observes nothing. From the diligent reseller's and manufacturer's perspectives, $\Pr(\theta = \theta_L) = \gamma = 1 - \Pr(\theta = \theta_H)$. All players are risk-neutral and act to maximize their expected profits. However, the salesperson is protected by limited liability and cannot be charged any fixed payment.

   (a) (5 points) Suppose the manufacturer integrates the reseller. In the two-layer manufacturer-reseller relationship, find the first-best equilibrium under complete information.

   Hint. As the manufacturer can observe the sales effort and the sales outcome is binary, it suffices for the manufacturer to design a fixed payment α, a sales bonus $\beta \in [0,1]$, and a specified effort level $a \ge 0$ that maximizes its expected profit while ensuring the salesperson's participation. Moreover, due to the salesperson's limited liability, the fixed payment is restricted to be nonnegative. If the salesperson accepts the offer, he must exert the effort level a so that he may earn $\alpha + \beta x$.

(b) (5 points) In the two-layer manufacturer-reseller relationship, find the second-best equilibrium under information asymmetry.

Hint. Now the manufacturer designs a menu of contracts $\{(\alpha_L, \beta_L), (\alpha_H, \beta_H)\}$. A type-$i$ salesperson will be induced to choose (α_i, β_i) and earn $\alpha_i + \beta_i x$. Again, the fixed payments and sales bonuses should all be nonnegative.

(c) (10 points) Suppose the manufacturer and reseller do not integrate and the manufacturer include the knowledgeable reseller in the supply chain, find the equilibrium. Please assume that the knowledgeable reseller and salesperson observe θ after the upstream two players sign the contract.

Hint. The manufacturer offers (u, v) and the reseller offers (α, β).

(d) (10 points) Suppose the manufacturer include the diligent reseller, find the equilibrium.

Hint. The manufacturer offers (u, v) and the reseller offers $\{(\alpha_i, \beta_i, a_i)\}_{i\in\{L,H\}}$.

(e) (10 points) Compare your results in the above four parts. highlight the sales effort and the manufacturer's expected profit, and provide economic intuitions.

4. (15 points) Find a real-world example of a three-layer supply chain. Try to explain why the supply chain must have three layers (i.e., why players cannot integrate) or suggest that a reduction of layers should happen in the future. If possible, try to list some sources of potential private information in the supply chain and conjecture about the impacts.[1]

References

Kung, L.-C., Y.-J. Chen. 2011. Monitoring the market or the salesperson? The value of information in a multilayer supply chain. *Naval Research Logistics* **58**(3) 743–762.

[1] What you are doing in this problem is similar to what you need to do in completing your final project. You need to first identify a scenario like this (no need to be a supply chain have three layers, of course). After it, it remains to solve for the equilibrium and interpret the results.