Information Economics

Fall 2013

Suggested Solution for Homework 6

1. (a) The buyer buys the offer (Q, P) gets $U^{\theta}(Q, P) = \theta Q - \frac{1}{2}Q^2 + \frac{1}{2}\theta^2 - P$ as his effective utility after the after-sales selection. The seller's problem is

$$\max_{P, Q, \underline{\theta}} \quad (1-\underline{\theta})P$$

s.t. $\underline{\theta} Q - \frac{1}{2}Q^2 + \frac{1}{2}\underline{\theta}^2 - P = 0$, where $\underline{\theta}$ is the lowest type that buys the offer. The problem is equivalent to $\max\limits_{Q, \underline{\theta}} (1-\underline{\theta})(\underline{\theta}Q - \frac{1}{2}Q^2 + \frac{1}{2}\underline{\theta}^2)$, which has $Q = \underline{\theta}$ at any optimal solution. Therefore, the problem reduces to

$\max\limits_{\underline{\theta}} (1-\underline{\theta})\underline{\theta}^2$, and the FOC requires the optimal $\underline{\theta} = \frac{2}{3}$. It then follows that $Q = \frac{2}{3}$, $P = \frac{4}{9}$, and the profit (revenue) is $\frac{4}{27}$.

(b) The solution we obtained above is the same as that of pricing by minutes. The restriction of offering a single contract under pricing by quantity limits the seller's ability of screening consumers. Therefore, the seller earns less under this restriction. As pricing by minutes also fails to screen consumers, it is not surprising that their results are identical.

2. Let $p = P_r(t_1 \mid L)$ and $q = P_r(t_1 \mid R)$. (a) Player C does not see the type.

(b) If player F plays (L, L), we have $p = \frac{1}{2}$ and $q \in [0, 1]$. For $p = \frac{1}{2}$, player C will play B when L is observed. When he observes R, he may play B for $q \geq \frac{2}{3}$ or play N for $q \leq \frac{2}{3}$. However, if he plays B, type-1 player F will deviate to play R. So a pooling equilibrium $((L, L), (B, N), (p = \frac{1}{2}, q \in [0, \frac{2}{3}]))$ is possible.

(c) If player F plays (R, R), we have $p \in [0, 1]$ and $q = \frac{1}{2}$. For $q = \frac{1}{2}$, player C will play N when R is observed. When he observes L, he will play B for any $p \in [0, 1]$. However, this will make both types of player F deviate to play L. Therefore, it is impossible for player F to play (R, R).

2 (d) If player F plays (L, R), we have $p=1$ and $q=0$, so player C plays (B, N). However, type-2 player F will deviate to play L, so this is impossible.

(e) If player F plays (R, L), we have $p=0$ and $q=1$, so player C plays (B, B). In this case, player F will not deviate, so $((R, L), (B, B), (0, 1))$ is a separating equilibrium.

(f) $((L, L), (B, N), (1, [0, \frac{2}{3}]))$ and $((R, L), (B, B), (0, 1))$.

(g) The theory predicts that a type-1 player F should result in either "L and B" or "R and B". 5 out of 6 type-1 player F really results in these two results, so the prediction seems to be accurate. The theory also predicts that a type-2 player F should result in either "L and N" or "L and B". However, only 2 out of 4 type-2 player F really results in these two results, so the prediction is not so satisfactory. This is probably because that players may be risk averse and playing R guarantees a positive payoff (though not high) for a type-2 player F. If there are more time, strategies should typically be more similar as players become "more rational".

3 (a) If the firm plays $(1, 0)$, the posterior belief will be $(p=1, q=0)$. Upon observing 0, the consumer will play N; upon observing 1, the consumer will buy if and only if $15-6r_H \geq 0$, i.e., $r_H \geq 0.4$. No matter the consumer buys or not, the reliable firm has no incentive to deviate, so $((1, 0), (B, N), (1, 0))$ is an equilibrium if $r_H \geq 0.4$ and $((1,0), (N, N), (1, 0))$ is an equilibrium if $r_H \in [0.2, 0.4]$.

(b) If the firm plays $(0, 1)$, the posterior belief will be $(p=0, q=1)$. Upon observing 1, the consumer will not buy; upon observing 0, the consumer will buy if and only if $20r_H - 11 \geq 0$, i.e., $r_H \geq 0.55$. If the consumer buys, the unreliable firm will deviate to offer no warranty, so the $(0, 1)$ strategy is not part of an equilibrium. If the consumer does not buy, no firm will deviate. Therefore, $((0,1), (N, N), (0, 1))$ is an equilibrium for $r_H \in [0.2, 0.55]$.