IM 7011: Information Economics

Lecture 7: Adverse Selection: Screening
Two-type Models: Taylor and Xiao (2009)

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Introduction

- In this lecture, we discuss Taylor and Xiao (2009).[1]
- While it is an application of the two-type model, it has:
  - Supply chain coordination.
  - Rebates and returns contracts.
  - Endogenous adverse selection.

[1] Taylor, T., W. Xiao. 2009. Incentives for Retailer Forecasting: Rebates vs. Returns. *Management Science* **55**(10) 1654–1669.

Road map

- **Introduction and model**.
- Integrated system.
- Rebates contracts.
- Returns contracts.

Demand forecasting

- Supply-demand mismatch is costly.
- Firms try to do **forecasting** to obtain demand knowledge.
- In a supply chain, typically the retailer does forecasting.
  - The manufacturer may only **induce** the retailer to forecast.
  - It is also the retailer that incurs the forecasting cost.
  - We shall study how the **forecasting cost** affects the supply chain.
- Is it always beneficial to induce forecasting?
  - Forecasting allows the supply chain to reduce supply-demand mismatch.
  - It also places the manufacturer at an **informational disadvantage**!
- If inducing forecasting is beneficial, when? How?

Contract formats

- Whether inducing/encouraging forecasting is beneficial depends on how the system profit is split.
  - The **contract format** between the manufacturer and retailer matters.
- Two kinds of contracts alters the retailer's decision of forecasting.
- Under a **rebates** contract, the manufacturer pays a bonus to the retailer for each sold unit.
  - A rebates contract provides a **lottery** to the retailer.
  - It **encourages** the retailer to forecast.
- Under a **returns** contract, the manufacturer buys back unsold units.
  - A returns contract provides an **insurance** to the retailer.
  - It **discourages** the retailer to forecast.
- Which contract format is more beneficial for the manufacturer?

Demand forecasting

- A manufacturer (he) sells to a retailer (she), who faces uncertain consumer demands.
- The unit production cost is c and unit retail price is p.
- Without forecasting, firms believe that the random demand $D_N \sim F_N$.
- The retailer may **forecast** with a forecasting cost k.
- If she forecasts, she obtains a **private** demand **signal** $S \in \{H, L\}$.
- With probability λ, she observes a favorable signal:
  - $S = H$ makes the retailer **optimistic**.
  - She believes that the market is good and the updated demand $D_H \sim F_H$.
- With probability $1 - \lambda$, she observes a unfavorable signal:
  - $S = L$ makes the retailer **pessimistic**.
  - She believes that the market is bad and the updated demand $D_L \sim F_L$.
- We assume that $F_H(x) \leq F_L(x)$ and $F_N(x) = \lambda F_H(x) + (1 - \lambda) F_L(x)$ for all $x \geq 0$. We also assume that $F(\cdot)$ is strictly increasing.
- Let $\bar{F}_S(x) := 1 - F_S(x)$, $S \in \{H, L, N\}$.

An example for demand forecasting

- As an example, suppose that $D_L \sim \text{Uni}(0,1)$ and $D_H \sim \text{Uni}(0,2)$, i.e.,

$$F_L(x) = \begin{cases} x & \forall x \in [0,1] \\ 1 & \forall x \in (1,2] \end{cases} \quad \text{and} \quad F_H(x) = \frac{x}{2} \quad \forall x \in [0,2].$$

  - The market is either good or bad. If it is good, the demand is D_H. Otherwise, it is D_L.
- We may say that the demand $D(\theta) \sim \text{Uni}(0,\theta)$, where $\theta \in \{1,2\}$.
- The firms both believe that $\Pr(\theta = 2) = \lambda = 1 - \Pr(\theta = 1)$.
- Without knowing θ, a firm can only believe that the demand is $D_N \sim F_N = \lambda F_H + (1-\lambda) F_L$.

  - If the retailer forecasts, she knows θ and thus whether it is D_H or D_L.

Contractual terms: rebates contracts

- By offering a rebates contract, the manufacturer specifies a three-tuple

$$(q, r, t).$$

  - q is the order **quantity**.
  - r is the sales **bonus** per unit sales.[2]
  - t is the **transfer** payment.
- If the retailer accepts the contract, she pays t to purchase q units and the rebate r.
- Note that the manufacturer is not restricted to sell the products at a wholesale price.
  - If this is the case, he will specify (q, r, w) where $t = wq$.
  - To find the optimal rebates contract, such a restriction should not exist.
  - t may depend on q and r in any format.

[2] Note that this is a *linear rebate*. *Target rebates* are not discussed in this paper.

Contractual terms: returns contracts

- By offering a rebates contract, the manufacturer specifies a three-tuple

$$(q, b, t).$$

  - q is the order **quantity**.
  - b is the **buy-back price** per unit of unsold products.[3]
  - t is the **transfer** payment.
- If the retailer accepts the contract, she pays t to purchase q units and the buy-back price b.
- The manufacturer is still not restricted to sell the products at a wholesale price.
  - t may depend on q and b in any format.

[3]Note that all unsold products can be returned. *Partial returns* are not discussed in this paper.

The manufacturer's contract design problem

- Note that we assume that the manufacturer can offer a **take-it-or-leave-it** contract.
  - The retailer cannot choose quantities at her disposal.
  - She can only **accept of reject** the contract.
  - Her information makes her accept-or-reject decision more accurate.
- If the retailer does not forecast, a single contract is enough.
  - There is no information asymmetry.
  - Enough flexibility is ensured by the flexibility on t.
- If the retailer has private information (signal S), a **menu of contracts** should be offered to induce truth-telling.
  - As S is binary, a menu of two contracts is optimal.
- We assume that the manufacturer **cannot mix** rebates and returns.
  - We will see that mixing does not make the manufacturer better off.
- The retailer determines whether to obtain private information. This is a problem with **endogenous adverse selection**!

Timing

- The sequence of events is as follows:
  1. The manufacturer offers a (menu of) rebates or returns contract(s).
  2. The retailer decides whether to forecast. If so, she privately observes the demand signal.
  3. The retailer chooses a contract or reject the offer based on her signal.
  4. Demand is realized and payments are made.
- The manufacturer **can induce** the retailer to or not to forecast.
  - Whether the retailer forecasts is also private. However, the manufacturer can anticipate this.
- Alternative timing (not discussed in this paper):
  - The retailer forecasts after choosing a contract ($1 \to 3 \to 2 \to 4$).
  - The retailer forecasts before getting the offer ($2 \to 1 \to 3 \to 4$).

Research questions revisited

- Should the manufacturer induce the retailer to forecast?
- If so, how the manufacturer design the offer?
- Which type of contracts, rebates or returns, is more beneficial?
- Efficiency? Inefficiency? Incentives? Information?

Road map

- Introduction and model.
- **Integrated system**.
- Rebates contracts.
- Returns contracts.

Integrated system without forecasting

- As a benchmark, let's first analyze the first-best situation: integration.
  - The decisions: (1) forecasting or not and (2) production quantity.
  - These decisions will be compared to determine efficiency.
- Suppose the system chooses not to forecast, it solves

$$\Pi_N(q_N) := p\mathbb{E}\min(q_N, D_N) - cq_N.$$

The optimal quantity is $q_N^I = \bar{F}_N^{-1}(\frac{c}{p})$.

- The optimized expected system profit is $\Pi_N(q_N^I)$.

Integrated system with forecasting

- Suppose the system chooses to forecast, it solves

$$\begin{aligned}\Pi_F(q_H, q_L) := \quad & \lambda\Big[p\mathbb{E}\min(q_H, D_H) - cq_H\Big] \\ & + (1-\lambda)\Big[p\mathbb{E}\min(q_L, D_L) - cq_L\Big].\end{aligned}$$

  The optimal quantities are $q_S^I = \bar{F}_S^{-1}(\frac{c}{p})$, $S \in \{H, L\}$.
  - By observing different signals, the quantity can be **adjusted** accordingly.
  - If no adjustment, i.e., $q_H = q_L = q$, then forecasting brings **no benefit**:

$$\Pi_F(q, q) = \Pi_N(q) \quad \forall q \geq 0.$$

- The optimized expected system profit is $\Pi_F(q_H^I, q_L^I)$.

Integrated system: forecasting or not?

- If forecasting is free, the system should **always forecast**:

$$\Pi_F(q_H^I, q_L^I) \geq \Pi_F(q_N^I, q_N^I) = \Pi_N(q_N^I).$$

- However, forecasting requires a cost k.
  - Whether the system should forecast depends on the value of k.
- The **performance gap** $k^I := \Pi_F(q_H^I, q_L^I) - \Pi_N(q_N^I)$ is the threshold.

Proposition 1

If $k < k^I$, the system should forecast and produce q_H^I (q_L^I) upon observing signal H (L). Otherwise, the system should not forecast and should produce q_N^I.

Road map

- Introduction and model.
- Integrated system.
- **Rebates contracts**.
- Returns contracts.

Rebates contracts

- Here we study the manufacturer's optimal strategy for offering **rebates contracts**.
- He has two options:
  - Inducing the retailer to forecast.
  - Inducing the retailer not to forecast.
- We will first find the optimal contracts in either case. Then we make comparisons to obtain the manufacturer's optimal strategy.
- In all equilibria, the retailer will accept a contract. Let

$$R^r(S, C) := (p + r_C)\mathbb{E}\min(q_C, D_S) - t_C,$$

  be the retailer's expected profit when:
  - she observes **signal** $S \in \{N, H, L\}$ (N for no forecasting) and
  - she chooses **contract** (q_C, r_C, t_C), $C \in \{N, H, L\}$.

No forecasting

- Suppose the manufacturer wants to drive the retailer not to forecast.
  - He will offer a single contract (q_N, r_N, t_N).
- Among rebates contracts that induce no forecasting, which is optimal?
- By accepting (q_N, r_N, t_N) with no forecasting, the retailer earns

$$R^r(N, N) := (p + r_N)\mathbb{E}\min(q_N, D_N) - t_N.$$

- However, she may choose to forecast and then accept or reject the offer **based on her signal**. If she forecasts, the retailer earns

$$\lambda \max\{R^r(H, N), 0\} + (1 - \lambda) \max\{R^r(L, N), 0\} - k.$$

  - With probability λ she will observe $S = H$. She then determine whether to accept (and earn $R^r(H, N)$) or reject (and earn 0).
  - With probability $1 - \lambda$ she will observe $S = L$.
  - In both cases, she pays k for forecasting.

No forecasting: formulation

- To optimally induce no forecasting, the manufacturer solves

$$\begin{aligned} \max_{q_N, r_N, t_N} \quad & t_N - cq_N - r_N E \min\{q_N, D_N\} \\ \text{s.t.} \quad & R^r(N,N) \geq \lambda \max\{R^r(H,N), 0\} \\ & \qquad + (1-\lambda)\max\{R^r(L,N),0\} - k \\ & R^r(N,N) \geq 0. \end{aligned}$$

  - The first constraint ensures that the retailer prefers no forecasting.
  - The second constraint ensures that the retailer will participate.
  - **Incentives** are provided through contracts.
- Technical assumptions:
  - Naturally, $q_N \geq 0$ and $r_N \geq 0$ though not explicitly specified.
  - It is assumed that $t_N \in \mathbb{R}$. Money may transfer in **either direction**!

No forecasting: solution

Proposition 2

The optimal rebates contract inducing no forecasting is

k	q_N^*	r_N^*	t_N^*
$k \leq \Gamma(q_L^I)$	q_L^I	0	$pE\min(q_L^I, D_N) - \dfrac{\Gamma(q_L^I) - k}{1-\lambda}$
$k \in (\Gamma(q_L^I), \Gamma(q_N^I))$	$\Gamma^{-1}(k)$	0	$pE\min(\Gamma^{-1}(k), D_N)$
$k \geq \Gamma(q_N^I)$	q_N^I	0	$pE\min(q_N^I, D_N)$

where $\Gamma(q) := (1-\lambda)p\int_0^q \left[\bar{F}_N(x) - \bar{F}_L(x)\right]dx$ *is strictly increasing in* $q \in (q_L^I, q_N^I)$ *and thus* $F^{-1}(\cdot)$ *is well-defined over* $[\Gamma(q_L^I), \Gamma(q_N^I)]$.

- The optimal contract depends on k.
- It is ugly, but it can be found.

No forecasting: intuitions

k	q_N^*	r_N^*	t_N^*
$k \leq \Gamma(q_L^I)$	q_L^I	0	$pE\min(q_L^I, D_N) - \dfrac{\Gamma(q_L^I) - k}{1-\lambda}$
$k \in (\Gamma(q_L^I), \Gamma(q_N^I))$	$\Gamma^{-1}(k)$	0	$pE\min(\Gamma^{-1}(k), D_N)$
$k \geq \Gamma(q_N^I)$	q_N^I	0	$pE\min(q_N^I, D_N)$

- A rebate encourages forecasting so **no rebate** should be offered.
- A large quantity encourages forecasting so q **increases in** k.
  - When k is large, it is easy to induce no forecasting.
  - The manufacturer can implement the **efficient quantity** (q_N^I) and capture all the surplus by the transfer.
  - When k is moderate, it is not too hard to induce no forecasting.
  - The manufacturer captures all the surplus with a **reduced quantity**.
  - When k is small, it is hard to induce no forecasting.
  - The manufacturer must **leave some rents** to the retailer by reducing t.

No forecasting: intuitions

- The retailer is “advantageous” when k is small. Does that make sense?
- The retailer gets rents though she does not have private information.
  - The **threat** of obtaining private information can generate rents!
- The power of threat depends on k:
  - When k is large, the threat is **weak** (noncredible). The manufacturer can be mean to the retailer (and use the transfer to extract everything).
  - When k is small, the threat is **strong** (credible). The manufacturer must be generous to the retailer.
- We may verify that the manufacturer's expected profit increases in k.
  - This is true if, and only if, he is required to induce no forecasting.

Forecasting

- Suppose the manufacturer wants to induce forecasting.
  - The retailer will have the private demand signal.
  - A **menu** of two contracts $\{(q_H, r_H, t_H), (q_L, r_L, t_L)\}$ will be offered.
- Now the manufacturer must ensures four things:
  - Once the retailer forecasts, she will select the intended contract.
  - Selecting the intended contract leaves the retailer a nonnegative profit.
  - The retailer must prefer forecasting to no forecasting.
  - Forecasting leaves the retailer a nonnegative profit.

Forecasting: formulation

- To optimally induce forecasting, the manufacturer solves

$$
\begin{aligned}
\max_{\substack{(q_H, r_H, t_H)\\(q_L, r_L, t_L)}} \quad & \lambda\Big[t_H - cq_H - r_H E\min\{q_H, D_H\}\Big] \\
& + (1-\lambda)\Big[t_L - cq_L - r_L E\min\{q_L, D_L\}\Big] \\
\text{s.t.} \quad & R^r(H,H) \geq R^r(H,L), \quad R^r(L,L) \geq R^r(L,H) \\
& R^r(H,H) \geq 0, \quad R^r(L,L) \geq 0 \\
& \lambda R^r(H,H) + (1-\lambda) R^r(L,L) - k \geq R^r(N,H) \\
& \lambda R^r(H,H) + (1-\lambda) R^r(L,L) - k \geq R^r(N,L) \\
& \lambda R^r(H,H) + (1-\lambda) R^r(L,L) - k \geq 0
\end{aligned}
$$

  - The first two IC constraints ensure truth-telling after forecasting.
  - The next two IR constraints ensure participation after forecasting.
  - The last three IC and IR constraints ensure forecasting.

Forecasting: solution

Proposition 3

The optimal rebates contract inducing forecasting is

$$q_L^* = \operatorname*{argmax}_{q \geq 0} \left\{ p \int_0^q [\bar{F}_L(x) - \lambda \bar{F}_H(x)]dx - (1-\lambda)cq \right\}$$

$$r_L^* = 0$$

$$t_L^* = pE\min(q_L^*, D_L)$$

$$q_H^* = q_H^I$$

$$r_H^* = \frac{k}{\lambda(1-\lambda)\Delta(q_H^I)}$$

$$t_H^* = (p + r_H^*)E\min(q_H^*, D_H) - p\Delta(q_L^*) - \frac{k}{\lambda}$$

where $\Delta(q) := \mathbb{E}\Big[\min(q, D_H) - \min(q, D_L)\Big]$.

Forecasting: intuition

- Whenever we want to differentiate agents through contract design, we need to provide incentives for them to tell the truth.
- Who has the incentive to lie?
  - A retailer always **tends to claim** that the market is **bad** to get generous contracts.
  - The high-type retailer wants to pretend to be the low-type one.
- That is why we have $r_H^* > r_L^* = 0$ and $q_H^I = q_H^* > q_L^*$.
  - An optimistic retailer likes rebates and high quantity.
  - To prevent her from mimicking the low type, the manufacturer **cuts down** r_L^* and q_L^*.
  - **Efficiency at top**: $q_H^I = q_H^*$.
  - **Monotonicity**: $q_H^* > q_L^*$.
  - **No rent at bottom** can also be verified.
  - $r_L^* = 0$: There is no point to offer a rebate to the low-type retailer.

Inducing forecasting or not

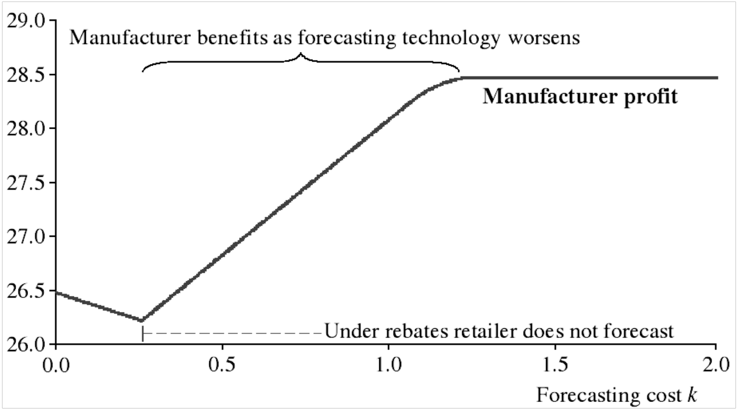
- We can find $\mathcal{M}_F^r(k)$ and $\mathcal{M}_N^r(k)$, the manufacturer's expected profit, as a function of k, when the retailer is induced to or not to forecast.
- Forecasting should be induced if and only if $\mathcal{M}_F^r(k) > \mathcal{M}_N^r(k)$.
- It can be verified that:
  - When $k = 0$, $\mathcal{M}_F^r(0) \geq \mathcal{M}_N^r(0)$: Inducing no forecasting is too costly when forecasting is free.
  - When k goes up, $\mathcal{M}_F^r(k)$ decreases (inducing forecasting becomes more costly) and $\mathcal{M}_N^r(k)$ increases (inducing no forecasting becomes easier).
- Therefore, there exists **a unique threshold** $k^r \geq 0$ such that

$$\mathcal{M}_F^r(k) > \mathcal{M}_N^r(k) \quad \Leftrightarrow \quad k < k^r.$$

  - Induce forecasting if and only if the **forecasting cost is low**.

Impact of the forecasting cost

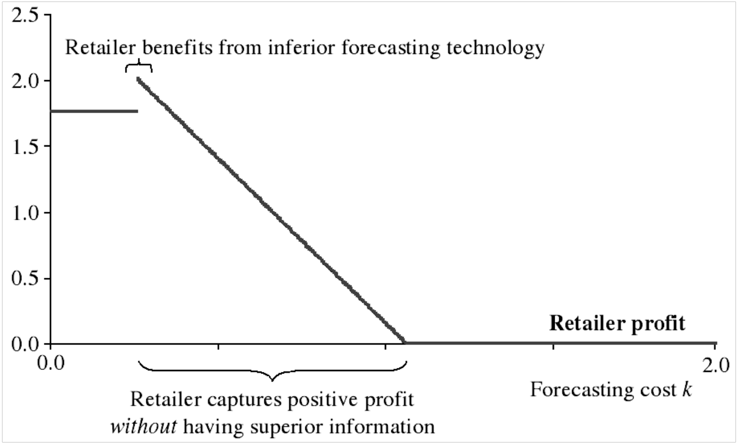
- The manufacturer may **prefer** a retailer with a **high** forecasting cost.



(Figure 1a in Taylor and Xiao (2009))

Impact of the forecasting cost

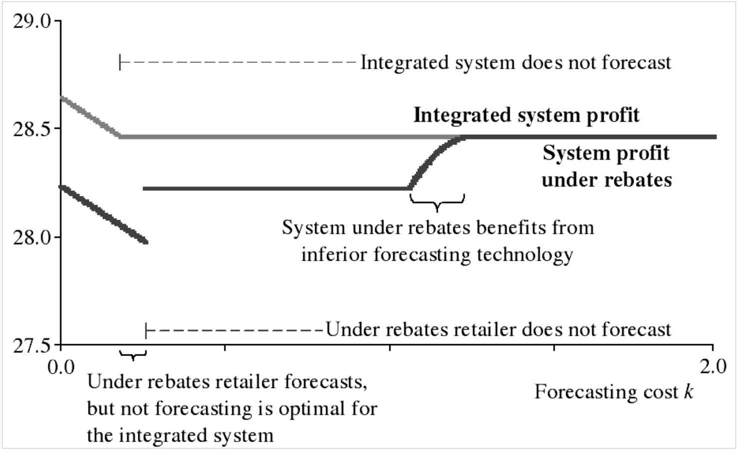
- The retailer may also **benefit** from a **high** forecasting cost.



(Figure 1b in Taylor and Xiao (2009))

Impact of the forecasting cost

- Rebates contracts **may not coordinate** the supply chain ($k^I \neq k^r$).
- The system may **benefit** from a **high** forecasting cost.



(Figure 1c in Taylor and Xiao (2009))

Summary for rebates contracts

- Manufacturers should not blindly seek out retailers with low forecasting cost.
  - It is easier for a better-forecasting retailer to get information advantage
- Retailers should not blindly reduce the forecasting cost.
  - Especially if the reduction crosses the threshold k^r.
- In practice, a manufacturer may reduce a retailer's forecasting cost.
  - He should do that only when the retailer is already good at forecasting.
- Note that all these conclusions are made when the manufacturer is restricted to rebates contracts.
  - How about returns contracts?
  - How about optimal contracts?

Road map

- Introduction and model.
- Integrated system.
- Rebates contracts.
- **Returns contracts**.

Returns contracts

- Here we study the manufacturer's optimal strategy for offering **returns contracts**.
- He may still chooses to induce the retailer to or not to forecast.
- In all equilibria, the retailer will accept a contract. Let

$$R^b(S, C) := p\mathbb{E}\min(q_C, D_S) + b_C\mathbb{E}\max(q_C - D_S, 0) - t_C,$$

be the retailer's expected profit when she observes signal $S \in \{N, H, L\}$ and chooses contract (q_C, b_C, t_C), $C \in \{N, H, L\}$.

No forecasting

- Suppose the manufacturer wants to drive the retailer not to forecast.
  - He will offer a single contract (q_N, b_N, t_N).
- Among returns contracts that induce no forecasting, which is optimal?
- Inducing the retailer not to forecast is surprisingly simple. Just provide a **full insurance**!
  - A contract satisfying $(q, b, t) = (q, p, pq)$ is a **full-returns** contract.[4]
  - Under a full-returns contract, the retailer has **no incentive to forecast**.
- The retailer **earns nothing** under a full-return contract.
- If the manufacturer offers the efficient quantity q^I, the manufacturer's expected profit is maximized to the expected system profit.
- The optimal returns contract is (q_N^I, p, pq_N^I).

[4] In Pasternack (1985), this is called a *full-credit* return contract.

Forecasting: formulation

- If the manufacturer wants to induce forecasting, he should offer a menu of two contracts $\{(q_H, b_H, t_H), (q_L, b_L, t_L)\}$.
- To optimally induce forecasting, the manufacturer solves

$$\begin{aligned}
\max_{\substack{(q_H, b_H, t_H)\\(q_L, b_L, t_L)}} \quad & \lambda\Big[t_H - cq_H - b_H E\max\{q_H - D_H, 0\}\Big] \\
& + (1-\lambda)\Big[t_L - cq_L - b_L E\max\{q_L - D_L, 0\}\Big] \\
\text{s.t.} \quad & R^b(H,H) \geq R^b(H,L), \quad R^b(L,L) \geq R^b(L,H) \\
& R^b(H,H) \geq 0, \quad R^b(L,L) \geq 0 \\
& \lambda R^b(H,H) + (1-\lambda)R^b(L,L) - k \geq R^b(N,H) \\
& \lambda R^b(H,H) + (1-\lambda)R^b(L,L) - k \geq R^b(N,L) \\
& \lambda R^b(H,H) + (1-\lambda)R^b(L,L) - k \geq 0
\end{aligned}$$

Forecasting: solution

- The optimal returns contract inducing forecasting is

$$
\begin{aligned}
q_L^* &= q_L^I \\
b_L^* &= p \\
t_L^* &= pq_L^I \\
q_H^* &= \max\{q_H^I, \Gamma^{-1}(k)\} \\
b_H^* &= 0 \\
t_H^* &= pE \min\left\{ \max\{q_H^I, \Gamma^{-1}(k)\}, D_H \right\} - k/\lambda
\end{aligned}
$$

  - The manufacturer should offer a **no-returns** (**full-returns**) contract for the optimistic (pessimistic) retailer.
  - **Efficiency at bottom**, not at top!
  - We need to discourage the retailer from doing no forecast but selecting (q_H^*, b_H^*, t_H^*). Upwards distorting q_H is effective: A retailer select a high-quantity contract only if she is optimistic enough.

Forecasting: surplus extraction

- It can be shown that the retailer still **earns nothing** when the manufacturer wants to induce forecasting.
- Why?
- The retailer may earn rents because she can **mimic** the low type when she is actually of the high type.
  - However, the full-returns contract leaves the retailer no surplus **regardless of her type**.
  - The manufacturer thus does not need to worry about the mimicking.
  - The retailer **has no informational advantage** even though she has private information!

Inducing forecasting or not

- Again, there is a unique threshold that determines whether the manufacturer should induce the retailer to forecast.
- (Most) surprisingly, the threshold is **always** identical to k^I, the threshold for the integrated system!

Proposition 4 (Proposition 6 in Taylor and Xiao (2009))

By offering a returns contract, manufacturer should induce forecasting if and only if $k < k^I$.

- *If* $k \geq k^I$, *a single full-returns contract is offered.*
- *If* $k < k^I$, *a full-returns contract and a no-returns contract are offered.*

In either case, the manufacturer's expected profit is the integrated system expected profit.

Inducing forecasting or not: intuition

- Full-returns contracts are too powerful!
- The manufacturer adopts the following strategy:
  - Always offer a full-returns contract to extract all the surplus from a type-N or type-L retailer.
  - Then the type-H also **loses her informational advantage**.
  - All I need to worry about is to induce forecasting when I should.
  - Offering a risky no-return contract with a large quantity encourages the retailer to forecast.
- **Screening** is not a problem. Inducing **information acquisition** is.
- However:
  - The retailer's threat of not to forecast is credible only if k is small.
  - But when k is small, the manufacturer prefers the retailer to forecast.
  - The threat is strong only when the manufacturer does not care about it.
- The key difference between rebates and returns is that **screening is a problem** when using rebates contracts.

Conclusions

- A supply chain in which the retailer may forecast or not is studied.
- Two types of contracts, rebates contracts and returns contracts, are analyzed and compared.
- From the manufacturer's perspective, returns contracts are better.
- In fact, returns contracts are optimal and coordinating.