Information Economics, Fall 2014
Suggested Solution for Homework 2

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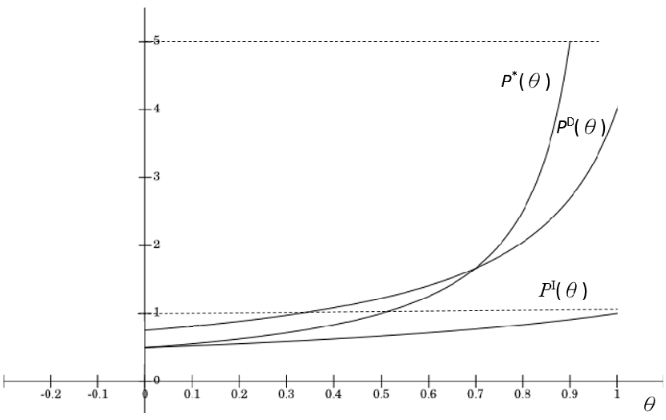
1. (a) We solve $\max_p 2p(1 - p + \theta p)$, whose optimal solution is $p^*(\theta) = \frac{1}{2(1-\theta)}$.

   (b) The three curves are shown in the figure. $p^*(\theta) > p^D(\theta)$ when θ is large, and $p^D(\theta) > p^*(\theta)$ when θ is small.



   (c) The only value of θ under which $p^D(\theta) = p^*(\theta)$ can be found by solving

$$\frac{2(3-\theta^2)}{(2-\theta)(4-\theta-2\theta^2)} = \frac{1}{2(1-\theta)} \Leftrightarrow 4 - 6\theta - \theta^2 + 2\theta^3 = 0.$$

   Numerically we may find a unique within-zero-and-one root 0.6991. And analytically we can show that the polynomial has exactly one root within zero and one.

   Note: When θ is small,decentralization not only drives the prices up. It makes the prices too high!

2. Full returns with full credits will always induce a too high equilibrium inventory level. To see this, note that if $R = 1$ and $r_2 = r_1$, the retailer will order Q_R^* such that $F(Q_R^*) = 1$ (from equation (7)). However, the channel-optimal quantity Q_T^* satisfies

$$F(Q_T^*) = \frac{p + g_2 - c}{p + g_2 - c_3} < 1,$$

   which implies $Q_R^* > Q_T^*$.

3. (a) The wholesale contract with the wholesale price w is a special case of the two-part tariff contract (w, t) with $t = 0$.

   (b) The retailer's expected profit can be formulated as

$$\pi_{\mathrm{R}}(q|w,t) = p\left\{ \int_0^q xf(x)dx + q[1 - F(q)] \right\} - (wq + t)$$

   if the retailer accepts the contract and chooses a quantity q. He should accept the contract if and only if $\pi_{\mathrm{R}}(q|w,t) \geq \pi_{\mathrm{R}}^*$, where π_{R}^* is the retailer's equilibrium expected profit under a wholesale contract.

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(c) The manufacturer's problem can be formulated as

$$\begin{aligned} \max_{w \geq 0, t} \quad & \pi_{\mathrm{M}}(w,t) = t + (w-c)q^* \\ \text{s.t.} \quad & q^* \in \arg\max_q \{\pi_{\mathrm{R}}(q|w,t)\} \\ & \pi_{\mathrm{R}}(q^*|w,t) \geq \pi_{\mathrm{R}}^*. \end{aligned}$$

(d) As long as the manufacturer set the wholesale price w to be the same as his cost c, the channel coordination can be achieved. Moreover, the whole system profit can be arbitrarily split by charging different fixed payment t. Win-win can thus be achieved.

4. (a) The two-part tariff (q,t) contract can be regarded as a wholesale contract for q units of the product with wholesale price $\frac{t}{q}$.

(b) The retailer's expected profit can be formulated as

$$\pi_{\mathrm{R}}(q,t) = p\left\{ \int_0^q xf(x)dx + q[1-F(q)] \right\} - t$$

if the retailer accepts the contract. He should accept the contract if and only if $\pi_{\mathrm{R}}(q,t) \geq \pi_{\mathrm{R}}^*$, where π_{R}^* is the retailer's equilibrium expected profit under a wholesale contract.

(c) The manufacturer's problem can be formulated as

$$\begin{aligned} \max_{q \geq 0, t} \quad & \pi_{\mathrm{M}}(q,t) = t - cq \\ \text{s.t.} \quad & \pi_{\mathrm{R}}(q,t) \geq \pi_{\mathrm{R}}^*. \end{aligned}$$

(d) As long as the whole system profit

$$p\left\{ \int_0^{q^*} xf(x)dx + q^*[1-F(q^*)] \right\} - cq^* \geq 0,$$

where q^* satisfies $1 - F(q^*) = \frac{c}{p}$, the system can generate a nonnegative profit by ordering the system-optimal quantity q^*. Then channel coordination can be achieved. For example, if the manufacturer offers q^* units with transfer

$$p\left\{ \int_0^{q^*} xf(x)dx + q^*[1-F(q^*)] \right\} = t^*,$$

then $\pi_{\mathrm{R}}(q^*, t^*) = 0$ and the retailer will accept the contract. Arbitrary profit spliting can also be achieved by lowering t^*.

5. (a) The worker solves $\max_{a \geq 0} t - \frac{1}{2}a^2$ and get the optimal service level $a^* = 0$. Having this in mind, the retailer solves $\max_{p,t} p(1-p) - t$ such that $t \geq 0$, where the constraint induces participation. The optimal solution is $t^* = 0$ and $p^* = \frac{1}{2}$. The retailer earns $\frac{1}{4}$ and the worker earns 0.

(b) After solving $\max_{a \geq 0} t + vp(1-p+a) - \dfrac{1}{2}a^2$, we derive the equilibrium service level $a^* = vp$ for the worker. And he earns $t + vp(1-p) + \frac{v^2p}{2}$.

(c) The retailer solves

$$\begin{aligned} \max_{p,t,v} \quad & p(1-p+vp)(1-v) - t \\ \text{s.t.} \quad & t + vp(1-p) + \tfrac{v^2p^2}{2} \geq 0. \end{aligned}$$

At optimality the constraint must be binding, so her problem can be reformulated to

$$\begin{aligned} & \max_{p,v} p(1-p+vp)(1-v) + vp(1-p) + \frac{v^2p^2}{2} \\ = & \max_{p,v} p - p^2 + vp^2 - \frac{v^2p^2}{2}. \end{aligned}$$

After solving the FOCs,

$$1 - 2p^* + 2vp^* - (v^*)^2 p^* = 0 \quad \text{and}$$
$$(p^*)^2 - v^*(p^*)^2 = 0,$$

we obtain the equilibrium retail price and commission rate $(p^*, v^*) = (1, 1)$ and equilibrium fixed payment $t^* = -\frac{1}{2}$. Then the retailer earns $\frac{1}{2}$ while the worker earns 0.

(d) It makes both players (at least weakly) better off. The retailer earns more while the worker remains the same.

(e) We solve $\max\limits_{p,a} p(1 - p + a) - \dfrac{1}{2}a^2$ with the FOCs $1 - 2p^* + a^* = 0$ and $p^* - a^* = 0$. We then obtain the efficient price and service level $p^* = a^* = 1$ with the whole system profit $\frac{1}{2}$.

(f) The contract with commission is efficient: The equilibrium price, service level, and system profit are all efficient. The reason is the following: The retailer can set $v = 1$ to induce the efficient service level and she will be willing to do that because the transfer t allows her to extract surplus from the worker.