Information Economics, Fall 2014
Suggested Solution for Homework 4

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1. Let $p = \Pr(t_1|\text{L})$ and $q = \Pr(t_1|\text{R})$.

   (a) The dashed lines state that player F's type is a hidden information, which cannot be seen by player C.

   (b) If player F plays (L, L), we have $p = \frac{1}{2}$ and $q \in [0, 1]$. For $p = \frac{1}{2}$, player C will play B when L is observed. When player C observes R, one the other hand, he will play B if $q \geq \frac{2}{3}$ or play N if $q \leq \frac{2}{3}$. However, if player C plays B in the right side, type-1 player F will deviate to play R. Thus, a pooling equilibrium ((L, L), (B, N), ($p = \frac{1}{2}, q \in [0, \frac{2}{3}]$)) is possible.

   (c) If player F plays (R, R), we have $p \in [0, 1]$ and $q = \frac{1}{2}$. For $q = \frac{1}{2}$, player C will play N when R is observed. When player C observes L, on the other hand, he will play B for any $p \in [0, 1]$. However, this will make both types of player F deviate to play L. Therefore, it is impossible for player F to play (R, R) in the equilibrium.

   (d) If player F plays (L, R), we have $p = 1$ and $q = 0$, and player C will play (B, N). However, type-2 player F will deviate to play L, which makes the separating equilibrium impossible.

   (e) If player F plays (R, L), we have $p = 0$ and $q = 1$, and player C will play (B,B). In this case both types of player F does not have an incentive to deviate, and thereby ((R,L), (B, B), (0, 1)) is a separating equilibrium.

   (f) ((L, L), (B, N), ($\frac{1}{2}$,$[0,\frac{2}{3}]$)) and ((R, L), (B, B), (0, 1)).

2. (a) If the firm plays (1, 0), the posterior belief will be ($p = 1, q = 0$). Upon observing 0, the consumer will play N; upon observing 1, the consumer will play B if and only if

   $$20r_H + 5(1 - r_H) - 11 = 15r_H - 6 \geq 0,$$

   i.e., $r_H \geq 0.4$ (from slide 16-18). No matter the consumer buys or not, the reliable firm has no incentive to deviate, so ((1, 0), (B, N), (1, 0)) is an equilibrium if $r_H \geq 0.4$ and ((1, 0), (N, N), (1, 0)) is an equilibrium if $r_H \in [0.2, 0.4]$.

   (b) If the firm plays (0, 1), the posterior belief will be ($p = 0, q = 1$). Upon observing 1, the consumer will not buy; upon observing 0, the consumer will buy if and only if

   $$20r_H - 11 \geq 0,$$

   i.e., $r_H \geq 0.55$. If the consumer buys, the unreliable firm will deviate to offer no warranty, so the (0, 1) strategy is not part of an equilibrium. If the consumer does not buy, no firm has an incentive to deviate. Therefore, ((0, 1), (N, N), (0, 1)) is an equilibrium for $r_H \in [0.2, 055]$.