IM 7011: Information Economics (Fall 2014)

The Screening Theory

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Road map

- **Introduction to screening**.
- First best with complete information.
- Incentives and the revelation principle.
- Finding the second best.
- Appendix: Proof of the revelation principle.

Principal-agent model

- Our introduction of **information asymmetry** will start here.
- We will study various kinds of **principal-agent** relationships.
- In the model, there is one **principal** and one or multiple **agents**.
  - The principal is the one that designs a mechanism/contract.
  - The agents act according to the mechanism/contract.
  - They are mechanism/contract **designers** and **followers**, respectively.
- It is also possible to have multiple principals competing for a single agent by offering mechanisms. This is the **common agency** problem.
- We will only discuss problems with one principal and one agent.

Asymmetric information

- There are two kinds of asymmetric information:
  - **Hidden information**, which causes the **adverse selection** problem.
  - **Hidden actions**, which cause the **moral hazard** problem.
- The principal may face two forms of adverse selection problems:
  - **Screening**: when the agent has private information.
  - **Signaling**: when the principal has private information.
- We have talked about the moral hazard problem.
- Today we discuss the screening problem.

Adverse selection: screening

- Consider the following buyer-seller relationship:
  - A manufacturer decides to buy a critical component of its product.
  - She finds a supplier that supplies this part.
  - Two kinds of technology can produce this component with **different unit costs**.
  - When a manufacturer faces the supplier, she **does not know** which kind of technology is owned by the supplier.
  - How much should the manufacturer pay for the part?
- The difficulty is:
  - If I know the supplier's cost is low, I will be able to ask for a low price.
  - However, if I ask him, he will **always** claim that his cost is high!
- The manufacturer wants to find a way to **screen** the supplier's **type**.

Adverse selection: screening

- An agent always want to **hide his type** to get bargaining power!
  - The "type" of an agent is a part of his **utility function** that is **private**.
- In the previous example:
  - The manufacturer is the principal.
  - The supplier is the agent.
  - The unit production cost is the agent's type.
- More examples:
  - A retailer does not know how to charge an incoming consumer because the consumer's **willingness-to-pay** is hidden.
  - An adviser does not know how to assign reading assignments to her graduate students because the students' **reading ability** is hidden.

Mechanism design

- One way to deal with agents' private information is to become more knowledgeable.
- When such an information-based approach is not possible, one way to screen a type is through **mechanism design**.
  - Or in the business world, **contract design**.
  - The principal will design a mechanism/contract that can "find" the agent's type.
- We will start from the easiest case: The agent's type has only two possible values. In this case, there are **two types** of agents.

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Monopoly pricing

- We will use a **monopoly pricing** problem to illustrate the ideas.
- Imagine that you produce and sell one product.
- You are the only one who are able to produce and sell this product.
- How would you price your product to maximize your profit?

Monopoly pricing

- Suppose the demand function is $q(p) = 1 - p$. You will solve

$$\pi^* = \max\ (1-p)p \quad \Rightarrow \quad p^* = \frac{1}{2} \quad \Rightarrow \quad \pi^* = \frac{1}{4}.$$

- Note that such a demand function means consumers' **valuation** (willingness-to-pay) lie uniformly within $[0, 1]$.
  - A consumer's utility is $v - p$, where v is his valuation.
- We may visualize the **monopolist's profit**:

Monopoly pricing

- Here comes a critic:
  - "Some people are willing to pay more, but your price is too low!"
  - "Some potential sales are lost because your price is too high!"
- His (useless) suggestion is:
  - "Who told you that you may set only one price?"
  - "Ask them how they like the product and charge differently!"
- Does that work?
- **Price discrimination** is impossible if consumers' valuations are completely hidden to you.
- If you can see the valuation, you will charge each consumer his valuation. This is **perfect price discrimination**.

Information asymmetry and inefficiency

- Let's visualize the monopolist's profit under perfect price discrimination:

- Information asymmetry causes **inefficiency**.
  - However, it **protects** the agent.
- Note that decentralization does not necessarily cause inefficiency. Here information asymmetry is the reason!

The two-type model

- In general, no consumer would be willing to tell you his preference.
- Consider the easiest case with valuation heterogeneity: There are **two** kinds of consumers.
- When obtaining q units by paying T, a **type-θ** consumer's utility is

$$u(q, T, \theta) = \theta v(q) - T.$$

  - $\theta \in \{\theta_{\mathrm{L}}, \theta_{\mathrm{H}}\}$ where $\theta_{\mathrm{L}} < \theta_{\mathrm{H}}$. θ is the consumer's **private** information.
  - $v(q)$ is strictly increasing and strictly concave. $v(0) = 0$.
  - A **high-type** (**type-H**) consumer's θ is θ_{H}.
  - A **low-type** (**type-L**) consumer's θ is θ_{L}.
  - The seller believes that $\Pr(\theta = \theta_{\mathrm{L}}) = \beta = 1 - \Pr(\theta = \theta_{\mathrm{H}})$.
- The unit production cost of the seller is c. $c < \theta_{\mathrm{L}}$.
- By selling q units and receiving T, the seller earns $T - cq$.
- How would you price your product to maximize your expected profit?

The two-type model with complete information

- Under complete information, the seller sees the consumer's type.
- Facing a type-H consumer, the seller solves

$$\max_{q_{\rm H}\geq 0, T_{\rm H} \text{ urs.}} \quad T_{\rm H} - cq_{\rm H}$$
$$\text{s.t.} \quad \theta_{\rm H} v(q_{\rm H}) - T_{\rm H} \geq 0.$$

- To solve this problem, note that the constraint must be **binding** (i.e., being an equality) at any optimal solution.
  - Otherwise we will increase $T_{\rm H}$.
  - Any optimal solution satisfies $\theta_{\rm H} v(q_{\rm H}) - T_{\rm H} = 0$.
  - The problem is equivalent to

$$\max_{q_{\rm H}\geq 0} \theta_{\rm H} v(q_{\rm H}) - cq_{\rm H}.$$

  - The FOC characterize the optimal quantity $\tilde{q}_{\rm H}$: $\theta_{\rm H} v'(\tilde{q}_{\rm H}) = c$.
  - The optimal transfer is $\tilde{T}_{\rm H} = \theta_{\rm H} v(\tilde{q}_{\rm H})$.

The two-type model with complete information

- For the type-i consumer, the **first-best** solution $(\tilde{q}_i, \tilde{T}_i)$ satisfies

$$\theta_i v'(\tilde{q}_i) = c \quad \text{and} \quad \tilde{T}_i = \theta_i v(\tilde{q}_i) \quad \forall i \in \{\mathrm{L}, \mathrm{U}\}$$

- The **rent** of the consumer is his surplus of trading.
- In either case, the consumer receives **no rent**!
- The seller extracts all the rents from the consumer.
- Next we will introduce the optimal pricing plan under information asymmetry and, of course, deliver some insights to you.

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Pricing under information asymmetry

- When the valuation is hidden, the first-best plan does not work.
  - You cannot make an offer (a pair of q and T) according to his type.
- How about offering a **menu** of two contracts, $\{(\tilde{q}_{\text{L}}, \tilde{T}_{\text{L}}), (\tilde{q}_{\text{H}}, \tilde{T}_{\text{H}})\}$, for the consumer to select?
- You **cannot expect** the type-i consumer to select $(\tilde{q}_i, \tilde{T}_i)$, $i \in \{\text{L}, \text{U}\}$!
  - Both types will select $(\tilde{q}_{\text{L}}, \tilde{T}_{\text{L}})$.
  - In particular, the type-H consumer will earn a **positive rent**:

$$\begin{aligned} u(\tilde{q}_{\text{L}}, \tilde{T}_{\text{L}}, \theta_{\text{H}}) &= \theta_{\text{H}} v(\tilde{q}_{\text{L}}) - \tilde{T}_{\text{L}} \\ &= \theta_{\text{H}} v(\tilde{q}_{\text{L}}) - \theta_{\text{L}} v(\tilde{q}_{\text{L}}) \\ &= (\theta_{\text{H}} - \theta_{\text{L}}) v(\tilde{q}_{\text{L}}) > 0. \end{aligned}$$

- It turns out that the first-best solution is not optimal under information asymmetry.

Incentive compatibility

- The first-best menu $\{(\tilde{q}_{\text{L}}, \tilde{T}_{\text{L}}), (\tilde{q}_{\text{H}}, \tilde{T}_{\text{H}})\}$ is said to be **incentive-incompatible**:
  - The type-H consumer has an incentive to **hide** his type and **pretend** to be a type-L one.
  - This fits our common intuition!
- A menu is **incentive-compatible** if different types of consumers will select different contracts.
  - An incentive-compatible contract induces **truth-telling**.
  - According to his selection, we can identify his type!
- How to make a menu incentive-compatible?

Incentive-compatible menu

- Suppose a menu $\{(q_{\mathrm{L}}, T_{\mathrm{L}}), (q_{\mathrm{H}}, T_{\mathrm{H}})\}$ is incentive-compatible.
  - The type-H consumer will select $(q_{\mathrm{H}}, T_{\mathrm{H}})$, i.e.,

$$\theta_{\mathrm{H}} v(q_{\mathrm{H}}) - T_{\mathrm{H}} \geq \theta_{\mathrm{H}} v(q_{\mathrm{L}}) - T_{\mathrm{L}}.$$

  - The type-L consumer will select $(q_{\mathrm{L}}, T_{\mathrm{L}})$, i.e.,

$$\theta_{\mathrm{L}} v(q_{\mathrm{L}}) - T_{\mathrm{L}} \geq \theta_{\mathrm{L}} v(q_{\mathrm{H}}) - T_{\mathrm{H}}.$$

- The above two constraints are called the **incentive-compatibility constraints** (IC constraints) or **truth-telling** constraints.
- If the seller wants to do business with both types, she also needs the **individual-rationality constraints** (IR constraints) or **participation** constraints:

$$\theta_i v(q_i) - T_i \geq 0 \quad \forall i \in \{\mathrm{L}, \mathrm{U}\}.$$

- The seller may offer an incentive-compatible menu. But is it optimal?

Inducing truth-telling is optimal

- Among all possible pricing schemes (or mechanisms, in general), some are incentive compatible while some are not.
  - The first-best menu is not.
  - An incentive compatible menu is.
- The **revelation principle** tells us "Among all **incentive compatible** mechanisms, at least **one is optimal**."[1]
  - We may restrict our attentions to incentive-compatible menus!
  - The problem then becomes tractable.
- Contributors of the revelation principle include three Nobel Laureates: James Mirrlees in 1996, and Eric Maskin and Roger Myerson in 2007.
  - There are other contributors.
  - Related works were published in 1970s.

[1] A nonrigorous proof is provided in the appendix.

Reducing the search space

- How to simplify our pricing problem with the revelation principle?
  - We only need to search among menus that can induce truth-telling.
  - Different types of consumers should select different contracts.
  - As we have only two consumers, two contracts are sufficient.
  - One is not enough and three is too many!
- The problem to solve is

$$\max_{q_{\mathrm{H}},T_{\mathrm{H}},q_{\mathrm{L}},T_{\mathrm{L}}} \quad \beta\Big[T_{\mathrm{L}} - cq_{\mathrm{L}}\Big] + (1-\beta)\Big[T_{\mathrm{H}} - cq_{\mathrm{H}}\Big] \tag{OBJ}$$

$$\text{s.t.} \quad \theta_{\mathrm{H}}v(q_{\mathrm{H}}) - T_{\mathrm{H}} \geq \theta_{\mathrm{H}}v(q_{\mathrm{L}}) - T_{\mathrm{L}} \tag{IC-H}$$

$$\theta_{\mathrm{L}}v(q_{\mathrm{L}}) - T_{\mathrm{L}} \geq \theta_{\mathrm{L}}v(q_{\mathrm{H}}) - T_{\mathrm{H}} \tag{IC-L}$$

$$\theta_{\mathrm{H}}v(q_{\mathrm{H}}) - T_{\mathrm{H}} \geq 0 \tag{IR-H}$$

$$\theta_{\mathrm{L}}v(q_{\mathrm{L}}) - T_{\mathrm{L}} \geq 0. \tag{IR-L}$$

  - The two IC constraints ensure truth-telling.
  - The two IR constraints ensure participation.
- Next we will introduce how to solve this problem.

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Solving the two-type problem

- Below we will introduce the standard way of solving the standard two-type problem[2]

$$\max_{q_{\mathrm{H}}, T_{\mathrm{H}}, q_{\mathrm{L}}, T_{\mathrm{L}}} \quad \beta\Big[T_{\mathrm{L}} - cq_{\mathrm{L}}\Big] + (1-\beta)\Big[T_{\mathrm{H}} - cq_{\mathrm{H}}\Big] \qquad \text{(OBJ)}$$

$$\text{s.t.} \quad \theta_{\mathrm{H}} v(q_{\mathrm{H}}) - T_{\mathrm{H}} \geq \theta_{\mathrm{H}} v(q_{\mathrm{L}}) - T_{\mathrm{L}} \qquad \text{(IC-H)}$$

$$\theta_{\mathrm{L}} v(q_{\mathrm{L}}) - T_{\mathrm{L}} \geq \theta_{\mathrm{L}} v(q_{\mathrm{H}}) - T_{\mathrm{H}} \qquad \text{(IC-L)}$$

$$\theta_{\mathrm{H}} v(q_{\mathrm{H}}) - T_{\mathrm{H}} \geq 0 \qquad \text{(IR-H)}$$

$$\theta_{\mathrm{L}} v(q_{\mathrm{L}}) - T_{\mathrm{L}} \geq 0. \qquad \text{(IR-L)}$$

- The key is that we want to **analytically** solve the problem.
  - With the analytical solution, we may generate some insights.

[2]Technically, we should also have nonnegativity constraints $q_{\mathrm{H}} \geq 0$ and $q_{\mathrm{L}} \geq 0$. To make the presentation concise, however, I will hide these two constraints.

Step 1: Monotonicity

- By adding the two IC constraints

$$\theta_{\mathrm{H}} v(q_{\mathrm{H}}) - T_{\mathrm{H}} \geq \theta_{\mathrm{H}} v(q_{\mathrm{L}}) - T_{\mathrm{L}}$$

  and

$$\theta_{\mathrm{L}} v(q_{\mathrm{L}}) - T_{\mathrm{L}} \geq \theta_{\mathrm{L}} v(q_{\mathrm{H}}) - T_{\mathrm{H}},$$

  we obtain

$$\begin{aligned} & \theta_{\mathrm{H}} v(q_{\mathrm{H}}) + \theta_{\mathrm{L}} v(q_{\mathrm{L}}) \geq \theta_{\mathrm{H}} v(q_{\mathrm{L}}) + \theta_{\mathrm{L}} v(q_{\mathrm{H}}) \\ \Rightarrow\ & (\theta_{\mathrm{H}} - \theta_{\mathrm{L}}) v(q_{\mathrm{H}}) \geq (\theta_{\mathrm{H}} - \theta_{\mathrm{L}}) v(q_{\mathrm{L}}) \\ \Rightarrow\ & v(q_{\mathrm{H}}) \geq v(q_{\mathrm{L}}) \\ \Rightarrow\ & q_{\mathrm{H}} \geq q_{\mathrm{L}}. \end{aligned}$$

- This is the **monotoniciy** condition: In an incentive-compatible menu, the high-type consumer consume more.
  - Intuition: The high-type consumer prefers a high consumption.

Step 2: (IR-H) is redundant

- (IC-H) and (IR-L) imply that (IR-H) is redundant:

$$\begin{aligned} \theta_{\text{H}} v(q_{\text{H}}) - T_{\text{H}} &\geq \theta_{\text{H}} v(q_{\text{L}}) - T_{\text{L}} && \text{(IC-H)} \\ &> \theta_{\text{L}} v(q_{\text{L}}) - T_{\text{L}} && (\theta_{\text{H}} > \theta_{\text{L}}) \\ &\geq 0. && \text{(IR-L)} \end{aligned}$$

- The high-type consumer earns **a positive rent**. Full surplus extraction is impossible under information asymmetry.
- The problem reduces to

$$\begin{aligned} \max_{q_{\text{H}}, T_{\text{H}}, q_{\text{L}}, T_{\text{L}}} \quad & \beta \Big[T_{\text{L}} - c q_{\text{L}} \Big] + (1 - \beta) \Big[T_{\text{H}} - c q_{\text{H}} \Big] && \text{(OBJ)} \\ \text{s.t.} \quad & \theta_{\text{H}} v(q_{\text{H}}) - T_{\text{H}} \geq \theta_{\text{H}} v(q_{\text{L}}) - T_{\text{L}} && \text{(IC-H)} \\ & \theta_{\text{L}} v(q_{\text{L}}) - T_{\text{L}} \geq \theta_{\text{L}} v(q_{\text{H}}) - T_{\text{H}} && \text{(IC-L)} \\ & \theta_{\text{L}} v(q_{\text{L}}) - T_{\text{L}} \geq 0. && \text{(IR-L)} \end{aligned}$$

Step 3: Ignore (IC-L)

- Let's "guess" that (IC-L) will be redundant and ignore it for a while.
  - Intuition: The low-type consumer **has no incentive** to pretend that he really likes the product.
  - We will verify that the optimal solution of the relaxed program indeed satisfies (IC-L).
- The problem reduces to

$$\begin{aligned} \max_{q_{\rm H}, T_{\rm H}, q_{\rm L}, T_{\rm L}} \quad & \beta\Big[T_{\rm L} - cq_{\rm L}\Big] + (1-\beta)\Big[T_{\rm H} - cq_{\rm H}\Big] && \text{(OBJ)} \\ \text{s.t.} \quad & \theta_{\rm H} v(q_{\rm H}) - T_{\rm H} \geq \theta_{\rm H} v(q_{\rm L}) - T_{\rm L} && \text{(IC-H)} \\ & \theta_{\rm L} v(q_{\rm L}) - T_{\rm L} \geq 0. && \text{(IR-L)} \end{aligned}$$

Step 4: Remaining constraints bind at optimality

$$\max_{q_\text{H}, T_\text{H}, q_\text{L}, T_\text{L}} \quad \beta\Big[T_\text{L} - cq_\text{L}\Big] + (1-\beta)\Big[T_\text{H} - cq_\text{H}\Big] \qquad \text{(OBJ)}$$

$$\text{s.t.} \quad \theta_\text{H} v(q_\text{H}) - T_\text{H} \geq \theta_\text{H} v(q_\text{L}) - T_\text{L} \qquad \text{(IC-H)}$$

$$\theta_\text{L} v(q_\text{L}) - T_\text{L} \geq 0. \qquad \text{(IR-L)}$$

- (IC-H) must be **binding** at any optimal solution:
  - The seller wants to increase T_H as much as possible.
  - She will keep doing so until (IC-H) is binding.
- (IR-L) must also be **binding** at any optimal solution:
  - The seller wants to increase T_L as much as possible.
  - She will keep doing so until (IR-L) is binding.
  - Note that increasing T_L makes (IC-H) more relaxed rather than tighter.
- Note that if we did not ignore (IC-L), i.e.,

$$\theta_\text{L} v(q_\text{L}) - T_\text{L} \geq \theta_\text{L} v(q_\text{H}) - T_\text{H},$$

then we cannot claim that (IR-L) is binding!

Step 5: Removing the transfers

- The problem reduces to

$$\max_{q_{\mathrm{H}}, T_{\mathrm{H}}, q_{\mathrm{L}}, T_{\mathrm{L}}} \quad \beta\Big[T_{\mathrm{L}} - cq_{\mathrm{L}}\Big] + (1-\beta)\Big[T_{\mathrm{H}} - cq_{\mathrm{H}}\Big] \qquad \text{(OBJ)}$$

$$\text{s.t.} \quad \theta_{\mathrm{H}} v(q_{\mathrm{H}}) - T_{\mathrm{H}} = \theta_{\mathrm{H}} v(q_{\mathrm{L}}) - T_{\mathrm{L}} \qquad \text{(IC-H)}$$

$$\theta_{\mathrm{L}} v(q_{\mathrm{L}}) - T_{\mathrm{L}} = 0. \qquad \text{(IR-L)}$$

- Therefore, we may remove the two constraints and replace T_{L} and T_{H} in (OBJ) by $\theta_{\mathrm{L}} v(q_{\mathrm{L}})$ and $\theta_{\mathrm{H}} v(q_{\mathrm{H}}) - \theta_{\mathrm{H}} v(q_{\mathrm{L}}) + \theta_{\mathrm{L}} v(q_{\mathrm{L}})$, respectively.
- The problem reduces to an **unconstrained** problem

$$\max_{q_{\mathrm{H}}, q_{\mathrm{L}}} \quad \beta\Big[\theta_{\mathrm{L}} v(q_{\mathrm{L}}) - cq_{\mathrm{L}}\Big] + (1-\beta)\Big[\theta_{\mathrm{H}} v(q_{\mathrm{H}}) - \theta_{\mathrm{H}} v(q_{\mathrm{L}}) + \theta_{\mathrm{L}} v(q_{\mathrm{L}}) - cq_{\mathrm{H}}\Big].$$

Step 6: Solving the unconstrained problem

- To solve

$$\max_{q_{\rm H}, q_{\rm L}} \beta\Big[\theta_{\rm L} v(q_{\rm L}) - c q_{\rm L}\Big] + (1-\beta)\Big[\theta_{\rm H} v(q_{\rm H}) - c q_{\rm H} - (\theta_{\rm H} - \theta_{\rm L}) v(q_{\rm L})\Big],$$

  note that because $v(\cdot)$ is strictly concave, the reduced objective function is strictly concave in $q_{\rm H}$ and $q_{\rm L}$.
- If $\frac{\theta_{\rm H} - \theta_{\rm L}}{\theta_{\rm H}} < \beta$, the **second-best** solution $\{(q_L^*, T_L^*), (q_H^*, T_H^*)\}$ satisfies the FOC:[3]

$$\theta_{\rm H} v'(q_H^*) = c \quad \text{and} \quad \theta_{\rm L} v'(q_L^*) = c\left[\frac{1}{1 - (\frac{1-\beta}{\beta}\frac{\theta_{\rm H} - \theta_{\rm L}}{\theta_{\rm L}})}\right].$$

[3] If $\frac{\theta_{\rm H} - \theta_{\rm L}}{\theta_{\rm H}} \geq \beta$, $q_L^* = 0$ and q_H^* still satisfies $\theta_{\rm H} v'(q_H^*) = c$.

Step 7: Verifying that (IC-L) is satisfied

- To verify that (IC-L) is satisfied, we apply

$$T_{\mathrm{L}} = \theta_{\mathrm{L}} v(q_{\mathrm{L}}) \quad \text{and} \quad T_{\mathrm{H}} = \theta_{\mathrm{H}} v(q_{\mathrm{H}}) - (\theta_{\mathrm{H}} - \theta_{\mathrm{L}}) v(q_{\mathrm{L}}).$$

- With this, (IC-L)

$$\theta_{\mathrm{L}} v(q_{\mathrm{L}}) - T_{\mathrm{L}} \geq \theta_{\mathrm{L}} v(q_{\mathrm{H}}) - T_{\mathrm{H}}$$

  is equivalent to

$$0 \geq -(\theta_{\mathrm{H}} - \theta_{\mathrm{L}}) \Big[v(q_{\mathrm{H}}) - v(q_{\mathrm{L}}) \Big].$$

  With the monotonicity condition, (IC-L) is satisfied.

Inefficient consumption levels

- Recall that the first-best consumption levels $\tilde{q}_{\text{L}}$ and $\tilde{q}_{\text{H}}$ satisfy

$$\theta_{\text{H}} v'(\tilde{q}_{\text{H}}) = c \quad \text{and} \quad \theta_{\text{L}} v'(\tilde{q}_{\text{L}}) = c.$$

Moreover, the second-best consumption levels satisfy

$$\theta_{\text{H}} v'(q_H^*) = c \quad \text{and} \quad \theta_{\text{L}} v'(q_L^*) = c\left[\frac{1}{1 - \left(\frac{1-\beta}{\beta}\frac{\theta_{\text{H}}-\theta_{\text{L}}}{\theta_{\text{L}}}\right)}\right] > c.$$

- The high-type consumer consumes the **first-best** amount.
- For the low-type consumer, $v'(\tilde{q}_{\text{L}}) = \frac{c}{\theta_{\text{L}}} < v'(q_L^*)$. As $v(\cdot)$ is strictly concave (so $v'(\cdot)$ is decreasing), $q_L^* < \tilde{q}_{\text{L}}$.
- The low-type consumer consumes less than the first-best amount.
  - Information asymmetry causes inefficiency.
  - The consumption will only decrease. It will not become larger. Why?

Cost of inducing truth-telling

- Regarding the consumption levels:
  - We have $q_L^* < \tilde{q}_\text{L}$. Why do we decrease q_L?
  - Recall that under the first-best menu, the high-type consumer pretends to have a low valuation and earns $(\theta_\text{H} - \theta_\text{L})v(\tilde{q}_\text{L}) > 0$.
  - Because he prefers a high consumption level, we must **cut down** q_L to make him **unwilling to lie**.
  - Inevitably, decreasing q_L creates inefficiency.
- Regarding the consumer surplus:
  - In equilibrium, the low-type consumer earns $\theta_\text{L} v(q_L^*) - T_L^* = 0$.
  - However, the high-type consumer earns

$$\theta_\text{H} v(q_H^*) - T_H^* = (\theta_\text{H} - \theta_\text{L})v(q_L^*) > 0.$$

  - The high-type consumer earns a positive **information rent**.
  - The agent earns a positive rent **in expectation**.
- Note that the high-type consumer's rent depends on q_L^*.
- Cutting down q_L^* is to cut down his information rent!

Summary

- We discussed a two-type monopoly pricing problem.
- We found the first-best and second-best mechanisms.
  - First-best: with complete information.
  - Second-best: under information asymmetry.
  - Thanks to the revelation principle!
- For the second-best solution:
  - **Monotonicity**: The high-type consumption level is higher.
  - **Efficiency at top**: The high-type consumption level is efficient.
  - **No rent at bottom**: The low-type consumer earns no rent.
- Information asymmetry **protects the agent**.
  - But it hurts the principal and social welfare.

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The idea of the revelation principle

- In general, the principal designs a mechanism for the agent(s).
  - The mechanism specifies a game rule. Agents act according to the rules.
- When agents have private types, there are two kinds of mechanisms.
- Under an **indirect mechanism**:
  - The principal specifies a function mapping agents' actions to payoffs.
  - Each agent, based on his type and his belief on other agents' types, acts to maximize his expected utilities.
- Under a **direct mechanism**:
  - The principal specifies a function mapping agents' **reported types** to actions and payoffs.
  - Each agent, based on his type and his belief on other agents' types, **reports a type** to maximize his expected utilities.
- If a direct mechanism can reveal agents' types (i.e., making all agents report truthfully), it is a **direct revelation mechanism**.

The idea of the revelation principle

Proposition 1 (Revelation principle)

Given any equilibrium of any given indirect mechanism, there is a direct revelation mechanism under which the equilibrium is equivalent to the given one: In the two equilibria, agents do the same actions.

- The idea is to "imitate" the given equilibrium.
- The given equilibrium specifies each agent's (1) strategy to map his type to an action and (2) his expected payoff.
- We may "construct" a direct mechanism as follows:
  - Given any type report (some types may be false), find the **corresponding actions and payoffs** in the given equilibrium as if the agents' types are really as reported.
  - Then assign **exactly those actions and payoffs** to agents.
- If the agents all report truthfully under the direct mechanism, they are receiving exactly what they receive in the given equilibrium. Therefore, under the direct mechanism no one deviates.