Information Economics

The Signaling Theory

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Road map

- **Introduction**.
- Bayesian updating.
- The first example.

Signaling

- We have studied two kinds of principal-agent relationship:
  - Screening: the agent has hidden information.
  - Moral hazard: the agent has hidden actions.
- Starting from now, we will study the third situation: **signaling**.
  - The **principal** will have hidden information.
- Both screening and signaling are adverse selection issues.

Origin of the signaling theory

- Akerlof (1970) studies the market of **used cars**.
  - The owner of a used car knows the **quality** of the car.
  - Potential buyers, however, do not know it.
  - The quality is hidden information observed only by the principal (seller).
- What is the issue?
  - Buyers do not want to buy "lemons".
  - They only pay a price for a used car that is "**around average**".
  - Owners of **bad** used cars are happy for selling their used cars.
  - Owners of **good** ones do not sell theirs.
  - Days after days... there are only bad cars on the market.
  - The "expected quality" and "average quality" become lower and lower.
- **Information asymmetry** causes **inefficiency**.
  - In screening problems, information asymmetry protects agents.
  - In signaling problems, information asymmetry **hurts everyone**.
- That is why we need platforms that suggest prices for used cars.

Origin of the signaling theory

- Spence (1973) studies the market of **labors**.
  - One knows her **ability** (productivity) while potential employers do not.
  - The "quality" of the worker is hidden.
  - Firms only pay a wage for "**around average**" workers.
  - **Low**-productivity workers are happy. **High**-productivity ones are sad.
  - Productive workers leave the market (e.g., go abroad). Wages decrease.
- What should we do? No platform can suggest wages for individuals!
- That is why we get **high education** (or study in good schools).
  - It is not very costly for a high-productivity person to get a higher degree.
  - It is **more costly** for a low-productivity one to get it.
  - By getting a higher degree (e.g., a master), high-productivity people **differentiate** themselves from low-productivity ones.
  - Getting a higher degree is **sending a signal**.
- This will happen (as an equilibrium) even if education itself **does not** enhance productivity!
  - Though this may not be a good thing, it seems to be true.
  - Think about **certificates**.

Signaling

- Signaling is for the principal to send a message to the agent to **signal the hidden information**.
  - Sending a message requires an **action** (e.g., getting a degree).
- For signaling to be effective, different types of principal should take different actions.
  - It must be **too costly** for a type to take a certain action.
- Other examples:
  - A manufacturer offers a **warranty** policy to signal the product reliability.
  - A firm sets a high **price** to signal the product quality.
  - "Full **refund** if not tasty".

Signaling games

- How to model and analyze a signaling game?
  - There is a principal and an agent.
  - The principal has a **hidden type**.
  - The agent cannot observe the type and thus have a **prior belief** on the principal's type.
  - The principal chooses an **action** that is observable.
  - The agent then forms a **posterior belief** on the type.
  - Based on the posterior belief, the agent **responds** to the principal.
- The principal takes the action to **alter** the agent's belief.
- An example:
  - A firm makes and sells a product to consumers.
  - The **reliability** of the product is hidden.
  - Consumers have a prior belief on the reliability.
  - The firm chooses between **offering a warranty or not**.
  - By observing the policy, the consumer **updates his belief** and make the purchasing decision accordingly.
- We need to model belief updating by **the Bayes' theorem**.

Road map

- Introduction.
- **Bayesian updating**.
- The first example.

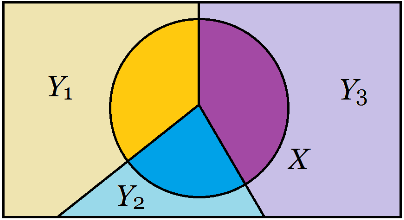
Law of total probability

- The following law is a component of Bayes' rule:

Proposition 1 (Law of total probability)

Let events Y_1, Y_2, ..., and Y_k be mutually exclusive and completely exhaustive and X be another event, then

$$\Pr(X) = \sum_{i=1}^{k} \Pr(Y_i) \Pr(X|Y_i).$$



Belief updating

- For some unknowns, we have some original estimates.
- We form a **prior belief** or assign a **prior probability** to the occurrence of an event.
  - Before I toss a coin, my belief of getting a head is $\frac{1}{2}$.
- If our estimation is accurate, the **relative frequency** of the occurrence of the event should be **close** to my prior belief.
  - In 100 trials, probably I will see 48 heads. $\frac{48}{100} \approx \frac{1}{2}$.
  - What if I see 60 heads? What if 90?
- In general, we expect observations to follow our prior belief.
- If this is not the case, we probably should update our prior belief into a **posterior belief**.

Example: Popularity of a product

- Suppose we have a product to sell.
- We do not know how consumers like it.
- Two possibilities (events): popular (P) and unpopular (U).
  - Our **prior** belief on P is 0.7.
  - We believe, with a 70% probability, that the product is popular.
- When one consumer comes, she may buy it (B) or go away (G).
  - If popular, the buying probability is 0.6.
  - If unpopular, the buying probability is 0.2.
- Suppose event G occurs once, what is our **posterior** belief?

Example: Popularity of a product

- We have the marginal probabilities $\Pr(P)$ and $\Pr(U)$:

	B	G	Total
P	?	?	0.7
U	?	?	0.3
Total	?	?	1

- We have the conditional probabilities:
  - $\Pr(B|P) = 0.6 = 1 - \Pr(G|P)$ and $\Pr(B|U) = 0.2 = 1 - \Pr(G|U)$.
- We thus can calculate those joint probabilities:

	B	G	Total
P	0.42	0.28	0.7
U	0.06	0.24	0.3
Total	?	?	1

Example: Popularity of a product

- We now can calculate the marginal probabilities $\Pr(B)$ and $\Pr(G)$:

	B	G	Total
P	0.42	0.28	0.7
U	0.06	0.24	0.3
Total	0.48	0.52	1

- Now, we observe one consumer going away (event G).
- What is the posterior belief that the product is popular (event P)?
  - This is the conditional probability $\Pr(P|G) = \frac{\Pr(P \cap G)}{\Pr(G)} = \frac{0.28}{0.52} \approx 0.54$.
- Note that we **update our belief** on P from 0.7 to 0.54.
- The fact that one goes away makes us **less confident**.
- If another consumer goes away, the updated belief on P becomes 0.37.
  - Use the old posterior as the new prior.
  - Use $\Pr(P|G)$ as $\Pr(P)$ and $\Pr(U|G)$ as $\Pr(U)$ and repeat.
- After five consumers go away in a row, the posterior becomes 0.07.
  - We tend to believe the product is unpopular!

Bayes' theorem

- By the law of total probability, we establish **Bayes' theorem**:

Proposition 2 (Bayes' theorem)

Let events Y_1, Y_2, ..., and Y_k be mutually exclusive and completely exhaustive and X be another event, then

$$\Pr(Y_j|X) = \frac{\Pr(Y_j \cap X)}{\Pr(X)} = \frac{\Pr(Y_j)\Pr(X|Y_j)}{\sum_{i=1}^{k}\Pr(Y_i)\Pr(X|Y_i)} \quad \forall j = 1, 2, ..., k.$$

- Sometimes we have events $\{Y_i\}_{i=1,\ldots,k}$ and X:
  - It is clear how Y_is affect X but not the other way.
  - Bayes' theorem is applied to **use X to infer $\{Y_i\}_{i=1,\ldots,k}$**.
- P and U naturally affect G and B but not the other way.
  - So we apply Bayes' theorem to use G to infer P and U:

$$\Pr(P|G) = \frac{\Pr(P)\Pr(G|P)}{\Pr(P)\Pr(G|P) + \Pr(U)\Pr(G|U)} = \frac{0.7 \times 0.4}{0.7 \times 0.4 + 0.3 \times 0.8} = 0.54.$$

Road map

- Introduction.
- Bayesian updating.
- **The first example**.

The first example

- A firm makes and sells a product with **hidden reliability** $r \in (0,1)$.
  - r is the probability for the product to be functional.
- If a consumer buys the product at price t:
  - If the product works, his utility is $\theta - t$.
  - If the product fails, his utility is $-t$.
- The firm may offer a **warranty** plan and repair a broken product.
  - The firm pays the repairing cost $k > 0$.
  - The consumer's utility is $\eta \in (0, \theta)$.
- The price is fixed (exogenous).
- Suppose $w = 1$ if a warranty is offered and 0 otherwise.
- Expected utilities:
  - The firm's expected utility is $u_F = t - (1-r)kw$.
  - The consumer's expected utility is $u_C = r\theta + (1-r)\eta w - t$.
- The consumer buys the product if and only if $u_C \geq 0$.
- The firm chooses whether to offer the warranty accordingly.

The first example: no signaling

- Suppose $r \in \{r_H, r_L\}$: The product may be reliable or unreliable.
  - $0 < r_L < r_H < 1$.
- Under complete information, the decisions are simple.
  - The firm's expected utility is $u_F = t - (1 - r_i)kw$.
  - The consumer's expected utility is $u_C = r_i\theta + (1 - r_i)\eta w - t$.
- Under incomplete information, they may make decision according to the **expected reliability**:
  - Let $\beta = \Pr(r = r_L) = 1 - \Pr(r = r_H)$ be the consumer's **prior belief**.
  - The expected reliability is $\bar{r} = \beta r_L + (1 - \beta) r_H$.
  - The firm's expected utility is $u_F = t - (1 - r_i)kw$.
  - The consumer's expected utility is $u_C = \bar{r}\theta + (1 - \bar{r})\eta w - t$.
- But wait! The **unreliable** firm will tend to offer **no warranty**.
  - Because $(1 - r_L)k$ is high.
  - This forms the basis of **signaling**.

The first example: signaling

- Below we will work with the following parameters:
  - $r_L = 0.2$ and $r_H = 0.8$.
  - $\theta = 20$ and $\eta = 5$.
  - $t = 11$ and $k = 15$.
- Payoff matrices (though players make decisions sequentially):

Consumer

Firm	Buy	Not
$w = 1$	$8, 6$	$0, 0$
$w = 0$	$11, 5$	$0, 0$

(Product is reliable)

Consumer

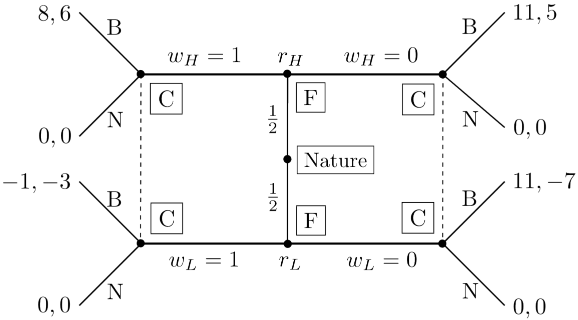
Firm	Buy	Not
$w = 1$	$-1, -3$	$0, 0$
$w = 0$	$11, -7$	$0, 0$

(Product is unreliable)

- The issue is: The consumer does not know **which matrix** he is facing!
- The reliable firm tries to convince the consumer that it is the first one.

Game tree

- We express this **game with incomplete information** by the following game tree:
  - F and C: players.
  - Nature: a fictitious player that draws the type randomly.
  - Let $\beta = \frac{1}{2}$ be the prior belief.

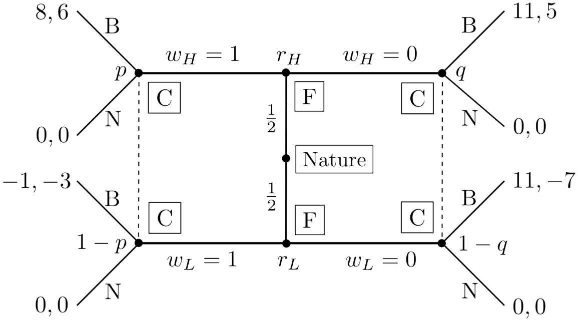


Concept of equilibrium

- What is a (pure-strategy) **equilibrium** in a signaling game?
- Decisions:
  - The "two" firms' actions: (w_H, w_L), $w_i \in \{0, 1\}$.
  - The consumer's strategy: (a_1, a_0), $a_j \in \{B, N\}$.
- Posterior beliefs:
  - Let $p = \Pr(r_H | w = 1)$ be the posterior belief upon observing a warranty.
  - Let $q = \Pr(r_H | w = 0)$ be the posterior belief upon observing no warranty.
- An equilibrium is a strategy-belief **profile** $((w_H, w_L), (a_1, a_0), (p, q))$:
  - No firm wants to deviate based on the consumer's posterior belief.
  - The consumer does not deviate based on his posterior belief.
  - The beliefs are updated according to the firms' actions by the Bayes' rule.
- It is extremely hard to "search for" an equilibrium. It is easier to "**check**" whether a given profile is one.
- We start from the firms' actions:[1]
  - Can $(1, 0)$ be part of an equilibrium? How about $(0, 1)$, $(1, 1)$, and $(0, 0)$?

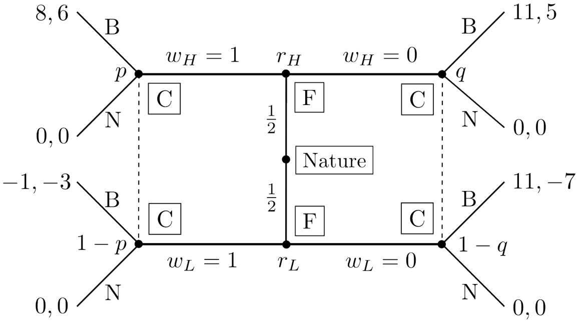
[1] It is typical to start from the principal's actions.

Warranty for the reliable product only



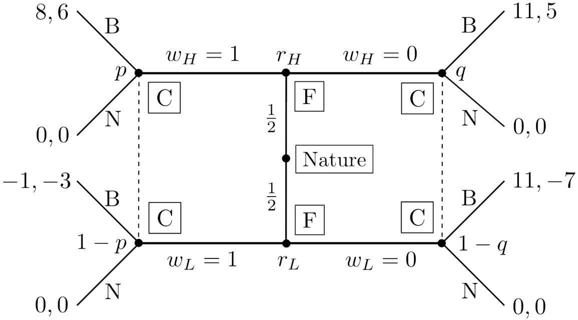
- We start from $((1, 0), (a_1, a_0), (p, q))$.
- Bayesian updating: $p = 1$, $q = 0$: $((1, 0), (a_1, a_0), (1, 0))$.
- Consumer $((1, 0), (B, N), (1, 0))$.
- No firm wants to deviate.

Warranty for the unreliable product only



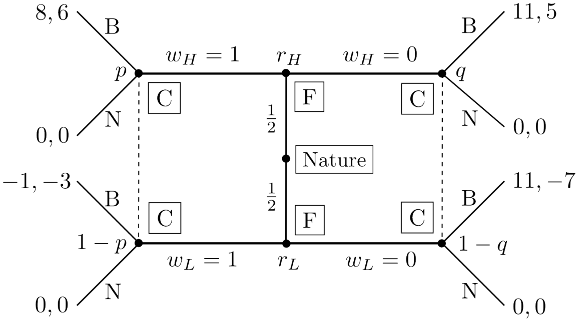
- We start from $((0, 1), (a_1, a_0), (p, q))$.
- Bayesian updating: $p = 0$, $q = 1$: $((0, 1), (a_1, a_0), (0, 1))$.
- Consumer: $((0, 1), (N, B), (0, 1))$.
- But now the unreliable firm deviates to $w_L = 0$!

Both offering warranties



- We start from $((1, 1), (a_1, a_0), (p, q))$.
- Bayesian updating: $p = \frac{1}{2}$, $q \in [0, 1]$: $((1, 1), (a_1, a_0), (\frac{1}{2}, [0, 1]))$.
- Consumer: $((1, 1), (B, \{B, N\}), (\frac{1}{2}, [0, 1]))$.
- If $a_0 = B$, no firm offers a warranty: $((1, 1), (B, N), (\frac{1}{2}, [0, 1]))$.
- But now the unreliable firm deviates to $w_L = 0$!

Both offering no warranty



- We start from $((0,0),(a_1,a_0),(p,q))$.
- Bayesian updating: $p \in [0,1]$, $q = \frac{1}{2}$: $((0,0),(a_1,a_0),([0,1],\frac{1}{2}))$.
- Consumer: $((0,0),(B,N),([\frac{1}{3},1],\frac{1}{2}))$, or $((0,0),(N,N),([0,\frac{1}{3}],\frac{1}{2}))$.
- For the former, the reliable firm deviates to $w_H = 1$. The latter is a pooling equilibrium.

Interpretations

- There are **pooling**, **separating**, and **semi-separating** equilibria:
  - In a pooling equilibrium, all types take the same action.
  - In a separating equilibrium, different types take different actions.
  - In a semi-separating one, some but not all types take the same action.
- In this example, there are two (sets of) equilibria:
  - A separating equilibrium $((1,0),(B,N),(1,0))$.
  - A pooling equilibrium $((0,0),(N,N),([0,\frac{1}{3}],\frac{1}{2}))$.
- What does that mean?

Interpretations

- The separating equilibrium is $((1,0),(B,N),(1,0))$:
  - The reliable product is sold with a warranty.
  - The unreliable product, offered with no warranty, is not sold.
  - The reliable firm **successfully signals** her reliability.
  - The system becomes more efficient.
  - Because it is too costly for the unreliable firm to do the same thing.
- The pooling equilibrium is $((0,0),(N,N),([0,\frac{1}{3}],\frac{1}{2}))$.
  - Both firms do not offer a warranty.
  - The consumer cannot update his belief.
  - The consumer does not buy the product.
- In this (and most) signaling game, there are **multiple** equilibria.