## Information Economics, Fall 2015 Pre-lecture Problems for Lecture 10

Instructor: Ling-Chieh Kung Department of Information Management National Taiwan University

Note. The deadline of submitting the pre-lecture problem is *9:20am*, *November 23, 2015*. Please submit a hard copy of your work to the instructor in class. Late submissions will not be accepted. Each student must submit her/his individual work. Submit ONLY the problem that counts for grades.

- 1. (0 points) Consider the binary outcome model introduced in the videos. Let the agent be risk-averse and action be observable.
  - (a) Given that  $w_1 = w_2$  at any optimal contract, let  $w = w_1 = w_2$  and reformulate the problem to a one with only two variables.
  - (b) Use the KKT condition to show that an optimal contract to the problem satisfies  $w_1 = w_2 = w$ , u(w) = a, and  $p'(a) = \frac{1}{u'(w)}$ .
- 2. (0 points) Consider the binary outcome model introduced in the videos. Let the action be unobservable. The optimality condition is derived in page 27 of the slides.
  - (a) Show that if the agent is risk-neutral, then  $u'(w_0) = u'(w_1) = 1$ .
  - (b) Show that if the agent is risk-neutral, then  $\mu = 0$ .
  - (c) Show that if the agent is risk-neutral, the unobservability of the action does not result in a moral hazard problem.
- 3. (10 points; 5 points each) Consider the LEN model introduced in the videos. Now, let the agent's cost of taking action a be  $\frac{1}{2}ca^2$  rather than  $\frac{1}{2}a^2$ , where c > 0 is exogenous.
  - (a) Given a contract (t, s), show that the optimal action that will be chosen by the agent is  $a^* = \frac{s}{c}$ . Make some comments on the impact of c on  $a^*$ .
  - (b) Show that the optimal contract satisfies  $s^* = \frac{1}{1+\eta c\sigma^2}$ . Make some comments on the impact of c on  $s^*$ .