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Suggested Solution for Problem Set 3

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1. (a) Let the decision variables be

$$x_i = \text{price of product } i,\ i = A, B.$$

The problem can then be formulated as

$$\begin{aligned} \max \quad & 1000x_A + 1500x_B \\ \text{s.t.} \quad & 10 - x_A \geq 0 && \text{(IR-1)} \\ & 15 - x_B \geq 0 && \text{(IR-2)} \\ & 10 - x_A \geq 8 - x_B && \text{(IC-1)} \\ & 15 - x_B \geq 12 - x_A. && \text{(IC-2)} \end{aligned}$$

The objective function maximizes the total sales revenue because group 1 members purchase A and group 2 members purchase B. Constraint (IR-1) ensures that a group 1 member is willing to buy product A. Constraint (IR-2) ensures that a group 2 member is willing to buy product B. Constraint (IC-1) ensures that a group 1 member prefers product A. Constraint (IC-2) ensures that a group 2 member prefers product B.

(b) First, note that (IR-2) is redundant because

$$15 - x_B \geq 12 - x_A \geq 10 - x_A \geq 0,$$

where the first inequality is (IC-2) and the last inequality is (IR-1). Once we remove (IR-2), we can show that (IC-2) is binding at any optimal solution. Suppose this is not the case, we will increase x_B for a sufficiently small amount without violating any constraint. We then have $x_B = x_A + 3$, which implies that (IC-1) is satisfied by any optimal solution because

$$8 - x_B = 5 - x_A \leq 10 - x_A.$$

Once we remove (IC-1), it is clear that (IR-1) must be binding at any optimal solution, so $x_A = 10$ and $x_B = 13$. These are the optimal prices.

2. (a) $\theta \in \{r_L, r_H\}$ is the retailer's type. $v(q)$ is the expected sales volume given the inventory level q, which is

$$\int_0^q xf(x)dx + \int_q^1 qf(x)dx = q - \frac{1}{2}q^2.$$

Within $[0, 1]$, it is clear that $v'(q) > 0$ and $v''(q) < 0$, and thus $v(q)$ is strictly increasing and strictly concave in the domain of interest.

(b) Our formula $\theta_i v'(q_i^{FB}) = c$ translates to $r_i(1 - q_i^{FB}) = c$, i.e., $q_i^{FB} = 1 - \frac{c}{r_i}$. The associated transfer is $t_i^{FB} = r_i v(q_i^{FB}) = \frac{1}{2r_i}(r_i^2 - c^2)$.

(c) The problem can be formulated as

$$\begin{aligned} \max \quad & \beta(t_L - cq_L) + (1-\beta)(t_H - cq_H) \\ \text{s.t.} \quad & r_L\left(q_L - \frac{1}{2}q_L^2\right) - t_L \geq r_L\left(q_H - \frac{1}{2}q_H^2\right) - t_H && \text{(IC-L)} \\ & r_H\left(q_H - \frac{1}{2}q_H^2\right) - t_H \geq r_H\left(q_L - \frac{1}{2}q_L^2\right) - t_L && \text{(IC-H)} \\ & r_L\left(q_L - \frac{1}{2}q_L^2\right) - t_L \geq 0 && \text{(IR-L)} \\ & r_H\left(q_H - \frac{1}{2}q_H^2\right) - t_H \geq 0 && \text{(IR-H)} \end{aligned}$$

(d) Because $\frac{r_H - r_L}{r_H} = \frac{1}{5} < \frac{1}{2}$, we have

$$r_H v'(q_H^*) = r_H(1 - q_H^*) = c \quad \Longleftrightarrow \quad q_H^* = 1 - \frac{2}{10} = \frac{4}{5}$$

and

$$r_L v'(q_L^*) = r_L(1 - q_L^*) = c\left(\frac{1}{1 - \frac{1-\beta}{\beta}\frac{r_H - r_L}{r_L}}\right) \quad \Longleftrightarrow \quad q_L^* = 1 - \frac{8/3}{8} = \frac{2}{3}.$$

The associated transfers are $t_L^* = 8(\frac{2}{3} - \frac{2}{9}) = \frac{32}{9}$ and $t_H^* = 10(\frac{12}{25} - \frac{4}{9}) + \frac{32}{9} = \frac{176}{45}$.

(e) We have $\frac{2}{3} < \frac{4}{5}$, which means $q_L^* < q_H^*$. Moreover, we have $\frac{2}{3} < 1 - \frac{2}{8} = \frac{3}{4}$, which mean $q_L^* < q_L^{FB}$.

3. (a) Consider the full returns contract $(q, b, t) = (q_N^I, p, q_N^I p)$. Since the manufacturer observes that the retailer does not forecast, his belief on demand is D_N and q_N^I maximizes system expected profit. Moreover, the retailer will accept the full returns contract with no rent. This implies that the manufacturer will offer the above full returns contract. Then the retailer earns 0 and the manufacturer earns $\pi_N(q_N^I)$.

(b) Again, consider the full returns contracts $(q_H, b_H, t_H) = (q_H^I, p, q_H^I p)$ and $(q_L, b_L, t_L) = (q_L^I, p, q_L^I p)$. We know this menu will give the manufacturer the highest possible profit as long as both types of retailer will choose the contract intended for her. It turns out that this is true, as the high-type consumer will earn 0 regardless of the contract she selects. Therefore, this is optimal. The manufacturer earns $\pi_F(q_H^I, q_L^I)$ and the ratiler earns 0. Note that her private information cannot protect her!

(c) As long as $k > 0$, the retailer should not forecast. If $k = 0$, forecasting or not does not matter.

4. In Pasternack (1985), offering full returns with full credits includes the retailer to order q such that $\Pr(D \leq q) = 1$, where D is the demand. This means the contract is too generous. However, as in Taylor and Xiao (2009) the retailer will be forced to order $q_S^I, S \in \{N, H, L\}$, the system-optimal quantity, such an overstocking situation will not occur. In short, it is because that the retailer chooses the order quantity in Pasternack (1985) but cannnot do so in Taylor and Xiao (2009), we see the difference.