

# Information Economics, Fall 2016

## Pre-lecture Problems 3

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**Note 1.** The deadline of submitting the pre-lecture problem is **9:20am, September 29, 2016**. Please submit a hard copy of your work into the instructor's mailbox in the first floor of Management Building 2. Late submissions will not be accepted. Each student must submit her/his individual work. Submit **ONLY** the problem that counts for grades.

**Note 2.** Please make your answer as clear (i.e., easy to read) as possible. We reserve the right to take away points when the correctness cannot be easily determined (e.g., when the writing is messy and cannot be easily understood).

1. (0 points) Recall the following Bertrand competition (for heterogeneous products): Two firms, 1 and 2, simultaneously set prices  $p_1$  and  $p_2$  for two substitutes. Given these prices, firm 1 sells  $q_1 = a - p_1 + bp_2$  and firm 2 sells  $q_2 = a - p_2 + bp_1$ , where  $a > 0$  and  $b \in [0, 1]$ . There is a unit production cost  $c < a$  for both firms. Suppose that each firm wants to maximize its own profit.
  - (a) Verify that the unique equilibrium is  $p_1^* = p_2^* = \frac{a-c}{2-b}$ .
  - (b) Show that when  $a = 1$  and  $c = 0$ , this result is the same as that in the I1 channel structure in McGuire and Staelin (1983).
2. (0 points) Consider the equilibrium wholesale prices  $w_i^*$  and retail prices  $p_i^*$  derived in pages 18 and 19 as functions of  $\theta$  (cf. equations (4-29) and (4-30) in McGuire and Staelin (1983)). Determine how they change when  $\theta$  changes. Make some economic interpretations.
3. (10 points; 2.5 points each) In lecture videos, we solved the static channel structure game

		M2	
		I	D
		$\frac{1}{(2-\theta)^2}$	$\frac{2+\theta}{4(2-\theta)(2-\theta^2)}$
M1	$\frac{1}{(2-\theta)^2}$	$\left[ \frac{4+\theta-2\theta^2}{2(2-\theta)(2-\theta^2)} \right]^2$	$\left[ \frac{4+\theta-2\theta^2}{2(2-\theta)(2-\theta^2)} \right]^2$
		$\left[ \frac{4+\theta-2\theta^2}{2(2-\theta)(2-\theta^2)} \right]^2$	$\frac{(2+\theta)(2-\theta^2)}{(2-\theta)(4-\theta-2\theta^2)^2}$
D	$\frac{2+\theta}{4(2-\theta)(2-\theta^2)}$	$\frac{(2+\theta)(2-\theta^2)}{(2-\theta)(4-\theta-2\theta^2)^2}$	$\frac{(2+\theta)(2-\theta^2)}{(2-\theta)(4-\theta-2\theta^2)^2}$

played by the two manufacturers. We showed that when  $0.708 < \theta < 0.931$ , this static game is actually a prisoners' dilemma: The two firms may be better off by choosing DD together, but II is the unique Nash equilibrium.

- (a) Set  $\theta = 0.8$  show that this game is indeed a prisoners' dilemma.
- (b) Set  $\theta = 0.95$  and show that there are two Nash equilibria.
- (c) Continue from Part (b). What if the game is played dynamically, i.e., manufacturer 1 first sets its channel structure and then manufacturer 2 makes its decision by observing manufacturer 1's decision? Does the not-so-good equilibrium go away or become more likely to happen?
- (d) Continue from Part (c). Does your conclusion hold for all  $\theta > 0.931$ ? Why or why not?

## References

McGuire, T. W., R. Staelin. 1983. An industry equilibrium analysis of downstream vertical integration. *Marketing Science* **2**(1) 115–130.