Linear Algebra and its Applications, Spring 2013
Homework 7

Instructor: Ling-Chieh Kung
Department of Information Management
National Taiwan University

Note 1. This homework is due ***8:30 am, October 29, 2013***. Please submit a hard copy into the homework box outside the TAs' lab.

Note 2. "Review exercises" should be found in the textbook (the fourth edition).

1. (5 points) Review Exercise 3.1.
2. (5 points) Review Exercise 3.2.
3. (5 points) Review Exercise 3.7.
4. (5 points) Review Exercise 3.8.
5. (10 points) Review Exercise 3.20.
6. (10 points) Review Exercise 3.21.
7. (10 points) Review Exercise 3.22.
8. (10 points) Review Exercise 3.24.
9. (15 points) Review Exercise 3.37.
10. (10 points) Suppose A has full row rank and x is in the row space of A, show that $A^+Ax = x$, where $A^+ = A^T(AA^T)^{-1}$ is the pseudo-inverse of A.
11. (15 points) Consider a given system $Ax = b$, which may or may not have a solution, and its normal equation $A^TAx = A^Tb$.
    (a) (5 points) If A has independent columns, show that $\hat{x}_1 = (A^TA)^{-1}A^Tb$ is a solution of the normal equation. Is it a unique solution?
    (b) (5 points) If A has independent rows, show that $\hat{x}_2 = A^+b = A^T(AA^T)^{-1}b$ is a solution of the normal equation. Is it a unique solution?
    (c) (5 points) If A has independent columns and independent rows, show that $\hat{x}_1 = \hat{x}_2$. How to express $\hat{x}_1$ and $\hat{x}_2$ by A and b?
    Note. There are at least two ways to show that $\hat{x}_1 = \hat{x}_2$, one relying on the existence of A^{-1} and the other one does not. Though you are not required to find both, would you try it?