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Homework 12

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1. In a dynamic game, player 1 first chooses between actions A and B. In either case, player 2 then chooses between actions C and D. The payoffs for players 1 and 2 are 4 and 1 under action combination (A, C), 3 and 2 under (A, D), 4 and 3 under (B, C), and 0 and 4 under (B, D).

   (a) Draw the game tree of this dynamic game.
   (b) What is player 2's best response function?
   (c) What is player 1's equilibrium strategy?
   (d) What is the equilibrium outcome?

2. A police officer caught two persons who broke a window of a general store. These two persons may have stolen some money from the store. Nevertheless, the police officer has no evidence. To induce the two persons confess, the officer may put them into separated rooms and create a prisoners' dilemma. Alternatively, he may simply put them in the same room. In this case, the two persons will coordinate (i.e., choose their actions together) and minimize the total number of months they are in prison. In either case, they will both be in prison for six months if both of them confess or one month if both of them deny. However, if one confesses and the other one denies, the former will be set free and the latter will be in prison for nine months. The officer tries to maximize the amount of time that he may put these two persons in prison. Completely analyze the three players' strategy, find their equilibrium behaviors, and find the equilibrium outcome.

3. In a small town, if there is only a Walmart store, Walmart earns 10 million dollars per year as profit. If there is only a Target store, Target earns 8 million dollars per year as profit. However, if there are both a Walmart store and a Target store, Walmart will lose 2 million and Target will lose 3 million dollars because the split demand is too small to compensate the fixed construction and operating costs. If one chooses not to build a store, it will earn nothing and lose nothing. Suppose currently no store has been built.

   (a) Suppose now Walmart has the chance to build a store in this small town before Target does. After Walmart makes the decision, Target can decide whether to build one or not. Will Walmart build a store? Will Target build a store?
   (b) Suppose now Target makes decision before Walmart does. Who will build a store, if any?

4. In a supply chain, there is a manufacturer, a distributor, and a retailer. The manufacturer produces the product at a unit production cost 0 and then chooses a unit wholesale price w to sell the product to the distributor. The distributor then chooses a resale price r to sell the product to the retailer. The retailer then chooses a retail price p to sell the product to end consumers. The consumer demand is $1 - p$. Each company aims to maximize its own profit.

   (a) What are the equilibrium wholesale, resale, and retail prices?
   (b) What are the three companies' profits in equilibrium?

5. In a supply chain, there is a manufacturer and two retailers. The manufacturer produces the product at a unit production cost c and then chooses a unit wholesale price w to sell the product to the two retailers. The two retailers are then engaged in a Cournot competition. Retailer 1 chooses a supply quantity q_1, retailer 2 chooses a supply quantity q_2, and the market clearing price is $a - b(q_1 + q_2)$. a, b, and c are all known positive constants. It is also known that $a > c$. The manufacturer can sell in total $q_1 + q_2$ units. Each company aims to maximize its own profit.

   (a) What are the wholesale price and market clearing price in equilibrium?
   (b) What are the two retailers' supply quantities and three companies' profits in equilibrium?