## Operations Research, Spring 2013 Suggested Solution for Homework 12

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1. (a) The game tree is depicted in Figure 1.

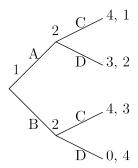


Figure 1: Game tree of Problem 1a

- (b) Player 2 should choose D if player 1 chooses A (because 2 > 1) and still choose D if player 1 choose B (because 4 > 3).
- (c) Knowing that player 2 will choose D in response to player 1's choice, player 1 will choose A to get a higher payoff (3 > 0).
- (d) The equilibrium outcome is (A, D): Player 1 chooses A and player 2 chooses D.
- 2. The police officer has two available actions: putting them in separated rooms or the same room. In the former case, the two persons will face a prisoners' dilemma and eventually confess. Each of them will be in prison for six months and the police officers' payoff is 12. In the latter case, the two persons will coordinate and choose to both deny. Each of them will be in prison for one month and the police officers' payoff is 2. Therefore, in equilibrium the officer will put them into different rooms and both of the two persons will confess.
- 3. (a) In Target's perspective, if Walmart builds a store, it will not build one; if Walmart does not build a store, it will build one. Knowing this, Walmart knows if it builds a store, it will be the only store in the town. Therefore, in equilibrium Walmart will build a store and Target will not.
  - (b) Following the same argument, Target will build a store and Walmart will not.
- 4. (a) In equilibrium, the wholesale price is  $\frac{1}{2}$ , the resale price is  $\frac{3}{4}$ , and the retail price is  $\frac{7}{8}$ .
  - (b) In equilibrium, the manufacturer earns  $\frac{1}{16}$ , the distributor earns  $\frac{1}{32}$ , and the retailer earns  $\frac{1}{64}$ .
- 5. (a) Given the wholesale price w, each retailer will order and supply  $\frac{a-w}{3b}$  units. The aggregate supply is  $\frac{2(a-w)}{3b}$ . Knowing this, the manufacturer will choose the wholesale price to be  $\frac{a+c}{2}$  to maximize her own profit. In equilibrium, the aggregate demand is  $\frac{a-c}{3b}$  and the market clearing price is  $\frac{2a+c}{3}$ .
  - (b) What are the two retailers' supply quantities and three companies' profits in equilibrium? The supply quantities are both  $\frac{a-c}{6b}$ . The manufacturer earns  $\frac{(a-c)^2}{6b}$ . Each of the two retailers earns  $\frac{(a-c)^2}{36b}$ .