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Suggested Solution for Project 2

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1. Let

$$
\begin{aligned}
&x_t = \text{production quantity in period } t,\ t = 1, ..., T,\\
&y_t = \text{ending inventory in period } t,\ t = 1, ..., T, \text{ and}\\
&z_t = \begin{cases} 1 & \text{if the machines are open in period } t \\ 0 & \text{otherwise} \end{cases} \quad , t = 1, ..., T
\end{aligned}
$$

be our decision variables. The complete formulation is

$$
\begin{aligned}
\min \quad & \sum_{t=1}^{T} P_t x_t + \sum_{t=1}^{T} S_t z_t + \sum_{t=1}^{T} H y_t \\
\text{s.t.} \quad & y_1 = I + x_1 - D_t \\
& y_t = y_{t-1} + x_t - D_t \quad \forall t = 2, ..., T \\
& x_t \leq \sum_{k=t}^{30} D_k z_t \quad \forall t = 1, ..., T \\
& x_t \geq 0,\ y_t \geq 0,\ z_t \in \{0, 1\} \quad \forall t = 1, ..., T.
\end{aligned}
$$

The objective function minimizes the total cost. The first two constraints ensure inventory balancing. The third constraint ensures that the machines are open if any positive amount is produced. Lastly, all variables are either nonnegative or binary.

2. Following from Problem 1, we define additional decision variables

$$
w_t = \begin{cases} 1 & \text{if the process is initiated in period } t \\ 0 & \text{otherwise} \end{cases} \quad , t = 1, ..., T
$$

and add the following new constraints

$$
\begin{aligned}
& w_1 \geq y_1 \\
& w_t \geq y_t - y_{t-1} \quad \forall t = 2, ..., T \\
& w_t \in \{0, 1\} \quad \forall t = 1, ..., T.
\end{aligned}
$$

The second constraint requires $w_t = 1$ if $y_t = 1$ and $y_{t-1} = 0$, which means the machines are open in period t but not in period $t - 1$. The first constraint handles the special case of period 1, which does not have a proceeding period. The last constraint ensures that w_ts are binary. We also need to add an additional term, the total initialization cost

$$
\sum_{t=1}^{T} R_t w_t,
$$

into the objective function.

3. Following from Problem 2, we add the following constraints

$$
\begin{aligned}
& w_t \leq y_t \quad \forall t = 1, ..., T - 2 \\
& w_t \leq y_{t+1} \quad \forall t = 1, ..., T - 2 \\
& w_t \leq y_{t+2} \quad \forall t = 1, ..., T - 2.
\end{aligned}
$$

These constraints ensure that, once $w_t = 1$, y_t, y_{t+1}, and y_{t+2} should all be 1. In other words, once a process is initiated in period t, the machines must be open in at least periods t, $t + 1$, and $t + 2$.

4. For this problem, we will formulate a completely new program. Let

$$\begin{aligned}
x_t &= \text{production quantity in period } t,\ t = 1, ..., T, \\
y_t &= \text{ending inventory "level" in period } t,\ t = 1, ..., T, \\
y_t^+ &= \text{ending inventory in period } t,\ t = 1, ..., T, \\
y_t^- &= \text{shortage in period } t,\ t = 1, ..., T, \text{ and} \\
z_t &= \begin{cases} 1 & \text{if the machines are open in period } t \\ 0 & \text{otherwise} \end{cases} \quad , t = 1, ..., T
\end{aligned}$$

be our decision variables. Here, inventory level may be positive or negative, where a positive inventory level means there are unsold produces and a negative one means there are unfulfilled demands. Our definition requires us to impose

$$y_t^+ = \max\{y_t, 0\} \quad \text{and} \quad y_t^- = \max\{-y_t, 0\}.$$

Note that the above constraints are nonlinear. We linearize them in the following complete formulation

$$\begin{aligned}
\min \quad & \sum_{t=1}^{T} P_t x_t + \sum_{t=1}^{T} S_t z_t + \sum_{t=1}^{T} H y_t^+ + \sum_{t=1}^{T} U y_t^- \\
\text{s.t.} \quad & y_1 = 50 + x_1 - D_1 \\
& y_t = y_{t-1} + x_1 - D_t \quad \forall t = 2, ..., T \\
& y_T = 0 \\
& y_t^+ \geq y_t,\ y_t^+ \geq 0 \quad \forall t = 1, ..., T \\
& y_t^- \geq -y_t,\ y_t^- \geq 0 \quad \forall t = 1, ..., T \\
& x_t \leq \left(\sum_{k=1}^{30} D_k - I \right) z_t \quad \forall t = 1, ..., T. \\
& x_t \geq 0,\ z_t \in \{0, 1\} \quad \forall t = 1, ..., T
\end{aligned}$$

The first two constraints ensure inventory balancing. The third constraint ensures that all the demands are fulfilled. The fourth and fifth constraints (together with the objective function) ensure that $y_t^+ = \max\{y_t, 0\}$ and $y_t^- = \max\{-y_t, 0\}$. The sixth constraint ensures that the machines are open if a positive amount is produced.[1] The last constraint ensures that x_ts are nonnegative and z_t are binary. Note that we should not impose nonnegativity on y_t! The objective function, in which the holding cost is measured with y_t^+ and the shortage cost is measured with y_t^-, now minimizes the total cost.

5. In using the MS Excel Solver, you probably have found that (1) the solution time is very long and (2) the solutions you obtain may be different. This is partly because the MS Excel Solver is not a very good one in solving mixed integer programs. I used another more professional solver to solve this problem and got my solution in the MS Excel file "ORSp13_project2_sol.xlsx", along with the MS Excel Solver program. The optimal objective value is $99,875.

[1] The upper bound of x_t is different from that in Part (a). Why?