# Operations Research, Spring 2014
# Homework 7

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**Note.** The deadline of this homework is ***1pm, 1 May, 2014***. Please put a hard copy of the work into the instructor's mailbox on the first floor of the Management Building II by the due time. Late submissions will not be accepted. Each student must submit her/his individual work.

1. (20 points; 5 points each) Each month, we use 1000 kilograms of sugar to make candies. Each time we order sugar from the supplier, it charges us a \$300 shipping cost every 500 kilograms. E.g., if we order 1200 kilograms, the shipping cost for the order will be \$900. The annual cost of holding a ton of sugar is \$30. Suppose the demand rate is constant and no shortage is allowed. Do an approximation by assuming that each month has exactly 30 days (so each year has exactly 360 days).
   (a) What is the optimal order quantity?
   (b) How many orders per year should be placed in average?
   (c) What is the optimal order cycle time?
   (d) If the order lead time is 20 days, what is the reorder point?
2. (10 points) Consider an EOQ problem with annual demand 500 units, unit holding cost \$1 per year, and unit ordering cost \$10. For $q \in [0, 300]$, depict $HC(q)$, $OC(q)$, and $TC(q)$, where they are the annual holding cost, annual ordering cost, and annual total cost, respectively.
3. (15 points; 5 points each) Consider an EOQ problem with annual demand $D$, unit holding cost $h$ per year, unit ordering cost $K$, and production rate $r$ units per year. Suppose we mistakenly believe that the annual demand is $2D$ and the unit holding cost is $\frac{h}{2}$.
   (a) What is the true EPQ $q^*$? What is the production quantity $q'$ that we will choose based on our wrong belief? What is $\frac{q'}{q^*}$?
   (b) Let $TC(q)$ be the true total cost given our chosen production lot size $q$. What is $\frac{TC(q')}{TCq^*}$?
   (c) Is EPQ a robust model? Briefly explain.
4. (10 points) Consider an EOQ problem with annual demand $D$, unit holding cost $h$ per year, and unit ordering cost $K$. Prove that the annual holding cost and annual ordering cost are identical if and only if the order quantity is the optimal order quantity.
5. (15 points; 5 points each) A hot dog vendor sells hot dogs for \$40 each at a night market. It costs \$10 for making a hot dog.
   (a) If the daily demand is uniformly distributed from 0 to 200, how many should he order?
   (b) If the daily demand is normally distributed with mean 150 and standard deviation 30, how many should he order?
   (c) If the daily demand is exponentially distributed with mean 50, how many should he order?
6. (10 points) A hot dog vendor sells hot dogs for \$40 each at a night market. It costs \$10 for making a hot dob. All the hot dogs he fails to sell by the midnight must be disposed at a unit disposal cost \$2. The daily demand is uniformly distributed from 0 to 200. When making hot dogs, it is required that the number of hot dogs made must be a multiple of 10. Find the optimal number of dogs to be prepared by the hot dog vendor.

7. (20 points; 5 points each) In a typical newsvendor problem, the distribution of the per period demand $D$ is not affected by the inventory level $q$. Given a problem setting, let the optimal quantity suggested by the newsvendor model be $q^*$. In practice, however, the distribution of $D$ may be affected by $q$. For each of the following four scenarios, first give an example that fits the scenarios. Then determine whether you should order fewer or more than the newsvendor quantity $q^*$. Intuitively explain why.

   (a) The experience of being rejected due to shortage will somehow discourage a consumer for coming to our store.

   (b) The rationing risk (the probability of being failed to buy a product before it is sold out) will encourage consumers to buy your product.

   (c) There is positive network externality: One feels happier when more people buy the product.

   (d) There is negative network externality: One feels happier when fewer people buy the product.