Operations Research
Lab Session

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Outline

1. Homework 2 illustration
2. Simplex method

Problem 4

x_{ij} = number of officers whose days off are on days i and j,
$i = 1, ..., 6, j = i + 1, ..., 7.$ Do not define redundant variables

$$\max \quad x_{12} + x_{23} + x_{34} + x_{45} + x_{56} + x_{67} + x_{17}$$

$$\text{s.t.} \quad x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} \leq 18$$

$$x_{12} + x_{23} + x_{24} + x_{25} + x_{26} + x_{27} \leq 10$$

$$x_{13} + x_{23} + x_{34} + x_{35} + x_{36} + x_{37} \leq 12$$

$$x_{14} + x_{24} + x_{34} + x_{45} + x_{46} + x_{47} \leq 8$$

$$x_{15} + x_{25} + x_{35} + x_{45} + x_{56} + x_{57} \leq 5$$

$$x_{16} + x_{26} + x_{36} + x_{46} + x_{56} + x_{67} \leq 5$$

$$x_{17} + x_{27} + x_{37} + x_{47} + x_{57} + x_{67} \leq 14$$

$$\sum_{i=1}^{6} \sum_{j=i+1}^{7} x_{ij} = 30$$

$$x_{ij} \geq 0 \quad \forall\ i = 1, ..., 6, j = i + 1, ..., 7.$$

Problem 6

x_{ij} = ounces of chemical j used to produce drug i,
$i = 1, ..., m$, $j = 1, ..., n$. D_i, S_j are just upper bound!

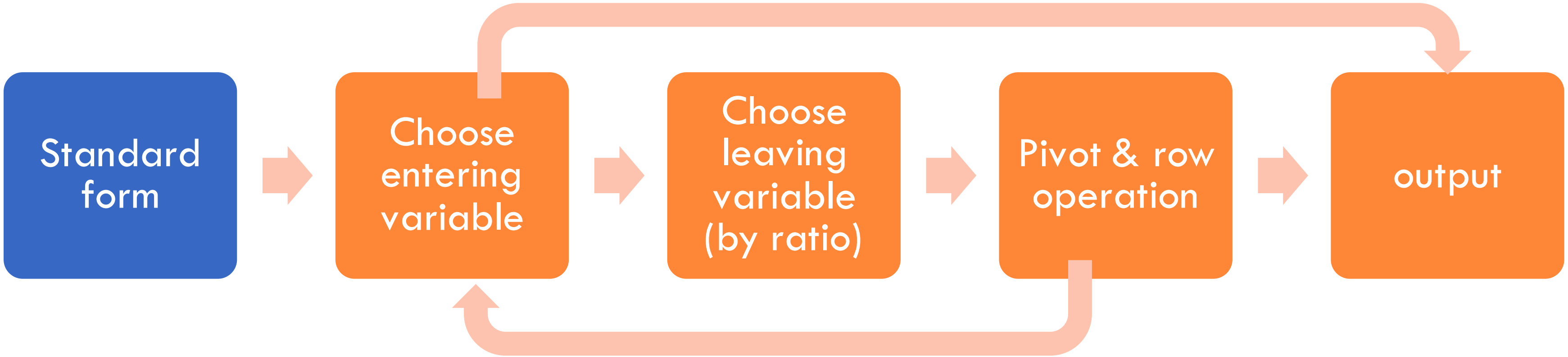
$$\max \quad \sum_{i=1}^{m} \sum_{j=1}^{n} P_i x_{ij} - \sum_{i=1}^{m} \sum_{j=1}^{n} C_j x_{ij}$$

$$\text{s.t.} \quad x_{ij} \geq M_{ij} \cdot \sum_{k=1}^{n} x_{ik} \quad \forall\, i = 1, ..., m, j = 1, ..., n$$

$$\sum_{j=1}^{n} x_{ij} \leq D_i \quad \forall\, i = 1, ..., m$$

$$\sum_{i=1}^{m} x_{ij} \leq S_j \quad \forall\, j = 1, ..., n$$

$$x_{ij} \geq 0 \quad \forall\, i = 1, ..., m, j = 1, ..., n.$$



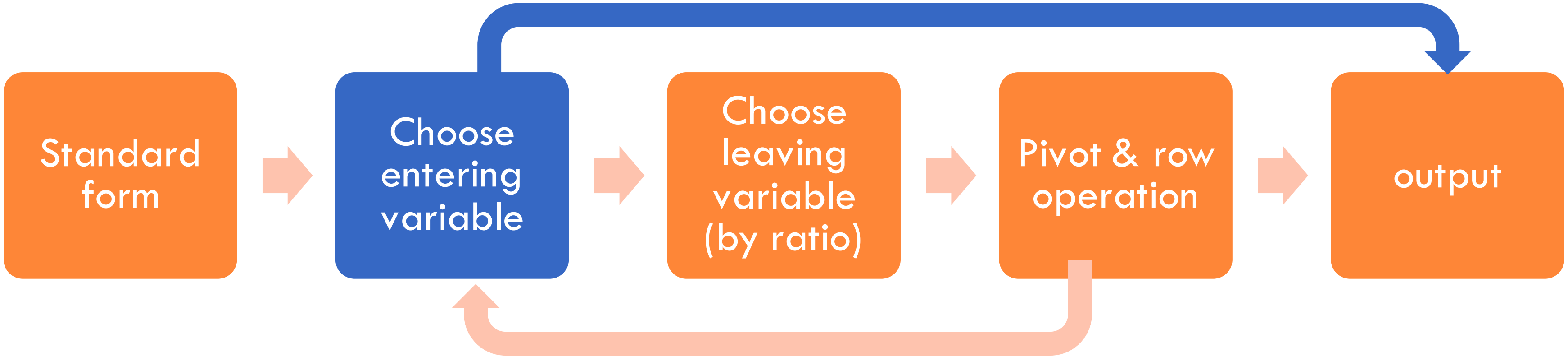
1. RHS nonnegative(in all steps)
2. Variable nonnegative
3. Constraint equity

Why we add negative sign to objected function?

max $2x_1 - x_2$

let $z = 2x_1 - x_2$

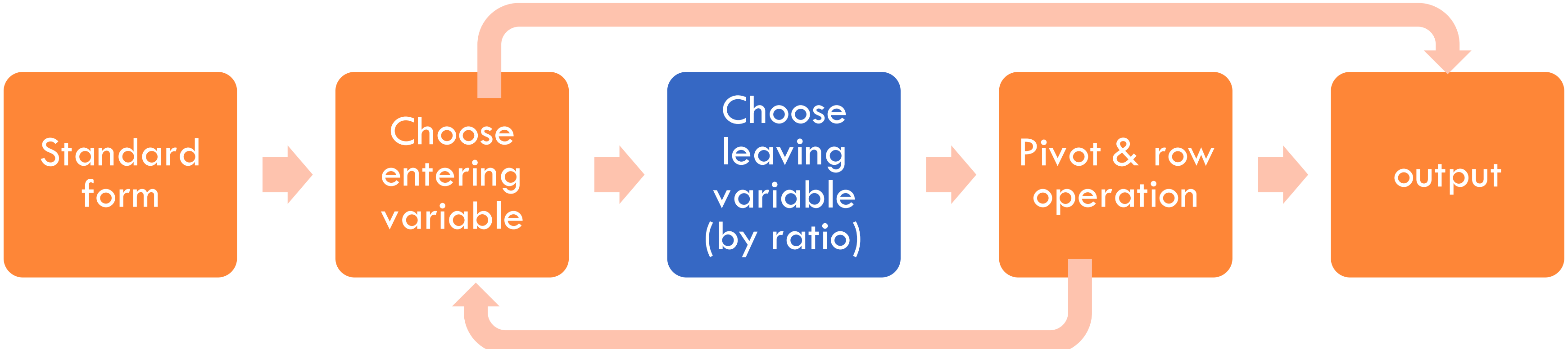
$z - 2x_1 + x_2 = 0$



1. Max => find negative
   Min => find positive

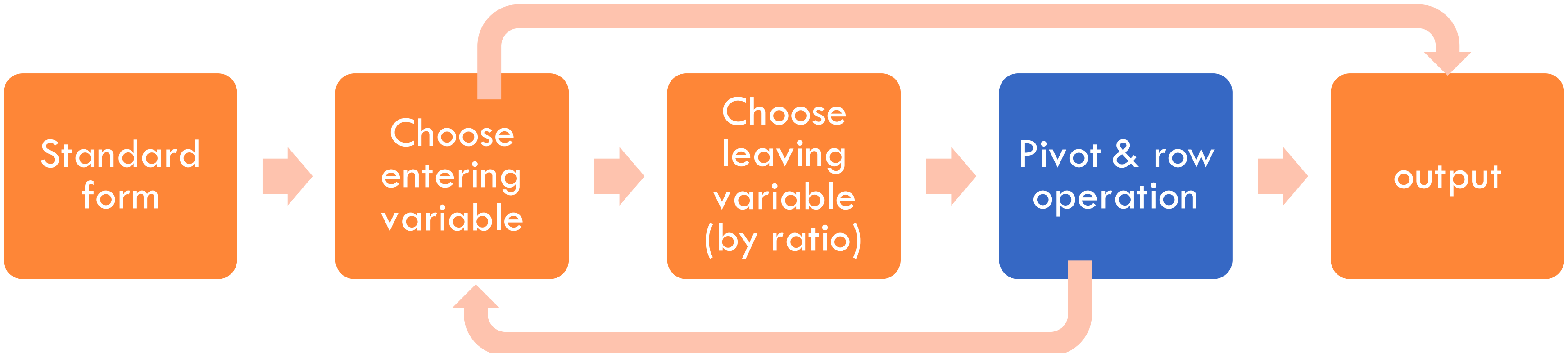
$$\text{Min } z - 2x_1 + x_2 = 0$$

2. If no one satisfy, you are in the optimum solution.



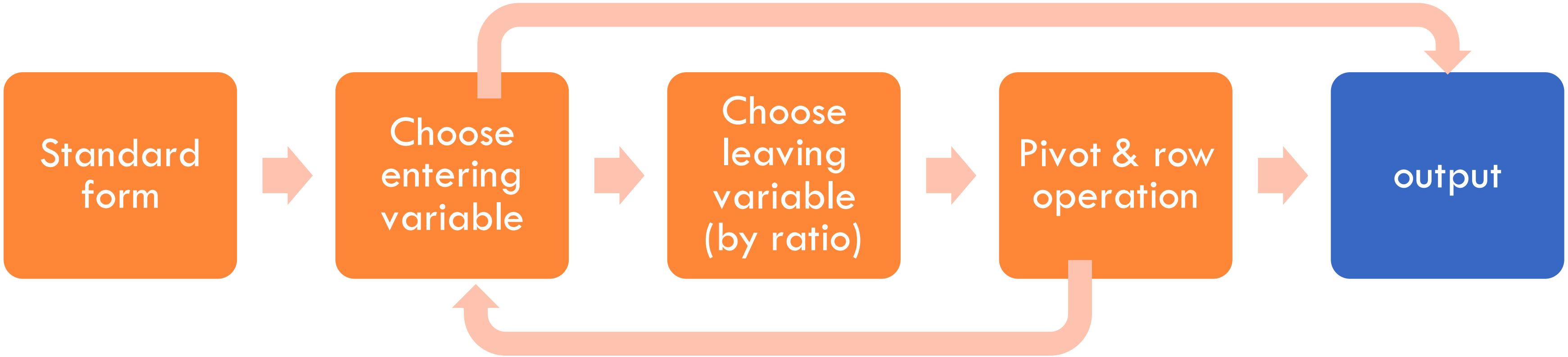
- Choose minimum positive ratio
- Can not choose negative
- Can not choose 0

-1	0	0	0	0	0	
-2	-1	1	0	0	$x_3 = 4$	-2
2	1	0	1	0	$x_4 = 8$	4
0	1	0	0	1	$x_5 = 3$	∞



$$
\begin{array}{cccc|c}
0 & 2 & 0 & -1 & -8 \\
\hline
0 & \boxed{\frac{3}{2}} & 1 & -\frac{1}{2} & x_3 = 2 \\
1 & \frac{1}{2} & 0 & \frac{1}{2} & x_1 = 4
\end{array}
$$

$$
\begin{array}{cccc|c}
0 & \boxed{0} & -\frac{4}{3} & -\frac{1}{3} & -\frac{32}{3} \\
\hline
0 & 1 & \frac{2}{3} & -\frac{1}{3} & x_2 = \frac{4}{3} \\
1 & \boxed{0} & -\frac{1}{3} & \frac{2}{3} & x_1 = \frac{10}{3}
\end{array}
$$



- Make sure there is no suitable entering variable
  Max => find negative
  Min => find positive
- Make sure your solution is reasonable

Practice

$$\begin{aligned} \text{Max } z = {} & 60x_1 + 30x_2 + 20x_3 \\ \text{s.t.} \quad & 8x_1 + 6x_2 + x_3 \le 48 \\ & 4x_1 + 2x_2 + 1.5x_3 \le 20 \\ & 2x_1 + 1.5x_2 + 0.5x_3 \le 8 \\ & x_i \ge 0 \quad \forall\, i = 1,2,3. \end{aligned}$$

solution

$z = 280,\ x_1 = 2,\ x_3 = 8$

(2,0,8,24,0,0)