Operations Research

Overview

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Welcome!

- This is an **introductory** Operations Research course designed for second-year students majoring in **Information Management**.
- My plan for today:
  - Ch. 1: What is Operations Research?
  - Syllabus and quiz.
  - An in-class planning game.
  - Ch. 2: Introduction to modeling.

What is Operations Research?

- **Operations Research** (OR) is:
  - the methodology to "**allocate** the available **resources** to the various activities in a way that is most effective for the organization as a whole."
  - "applied to problems that concern how to conduct and coordinate the **operations** (i.e., activities) within an organization."[1]
- It aims to **support decision making**.
  - Typical tools: intuitions, business senses, and experiences.
  - And OR (and other quantitative tools)!
  - By doing OR studies, we generate some suggestions to **decision makers**.

[1] Both quoted from *Introduction to Operations Research* by Hillier and Lieberman, the ninth edition.

Industry applications

- Important questions:
  - How to deliver 6.5 millions items to more than 220 countries each day?
  - In each region, where to build distribution hubs?
  - In each distribution hub, how to classify and sort items?
  - In each city, how to choose routes?
- What do you need?
  - Well-designed information systems.
  - Operations Research!
- Further reading:
  - The application vignette in Section 1.4.
  - The article on CEIBA with the complete story.

Industry applications

- Important questions:
  - How to determine the cities to connect?
  - How to schedule more than 2000 flights per day?
  - How to assign crews to flights?
  - How to reassign crews immediately when there is an emergency?
- What do you need?
  - Well-designed information systems.
  - Operations Research!
- Further reading:
  - The application vignette in Section 2.2.
  - The article on CEIBA with the complete story.

Applicability and limitations

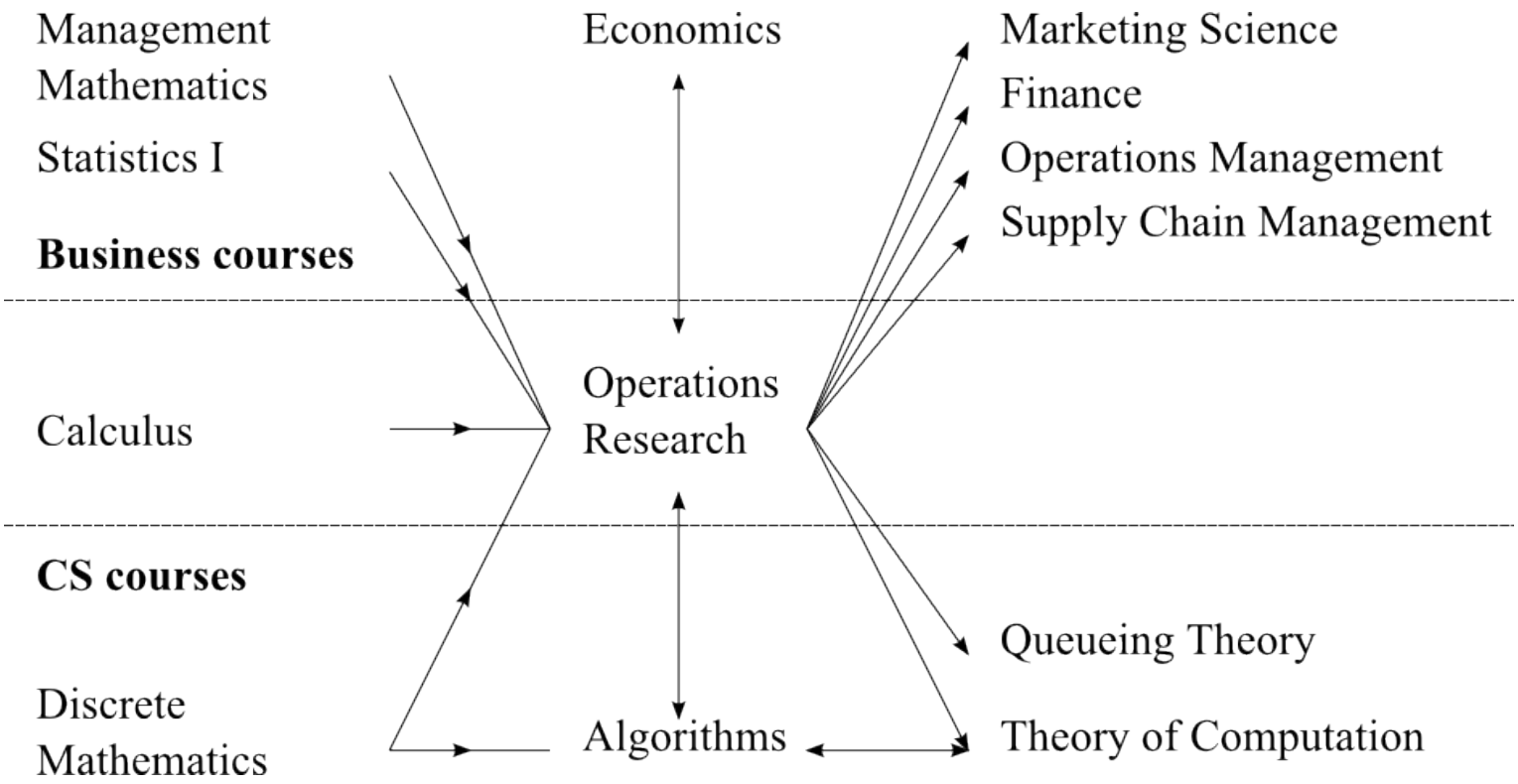
- It aims to support **decision making** in a **complicated** environment.
  - It is useless if we do not make decisions.
  - It is helpful if intuitions and experiences are not enough.
  - It is required if one's organization has many operations involved.
- It is a collection of **mathematical** (quantitative) methods.
  - Many methods come from **economics** and **computer science**.
  - It overlaps a lot with Management Science and Industrial Engineering.
- It is best for **quantifiable decisions**.
  - Those things that can be counted or measured.
  - E.g., quantities to produce, inventory to stock, amount to invest, routes to go, workers to assign, etc.
  - It is not so helpful for qualitative decisions.
- It almost always requires **computers**.
  - So that large-scale computations are possible.

In short...

- What is Operations Research?
- We use **engineering** approaches to solve **managerial** problems.
  - A field of applied mathematics for making **better business decisions**.

The role of OR in our IM department

- Operations Research is one of the few courses that lie in the **interface** between Business and Computer Science.



The role of OR in our IM department

- It is a promising direction if you:
  - Want to learn something that help you do business operations **without** requiring a lot of experiences and domain knowledge.
- It will also be very useful if you:
  - Will work on **mathematical problems** in Computer Science, Economics, Operations Management, Finance, and many other fields.
- For those of you who have not decided yet:
  - Study it so that you will not miss a chance in the future.

Before we start...

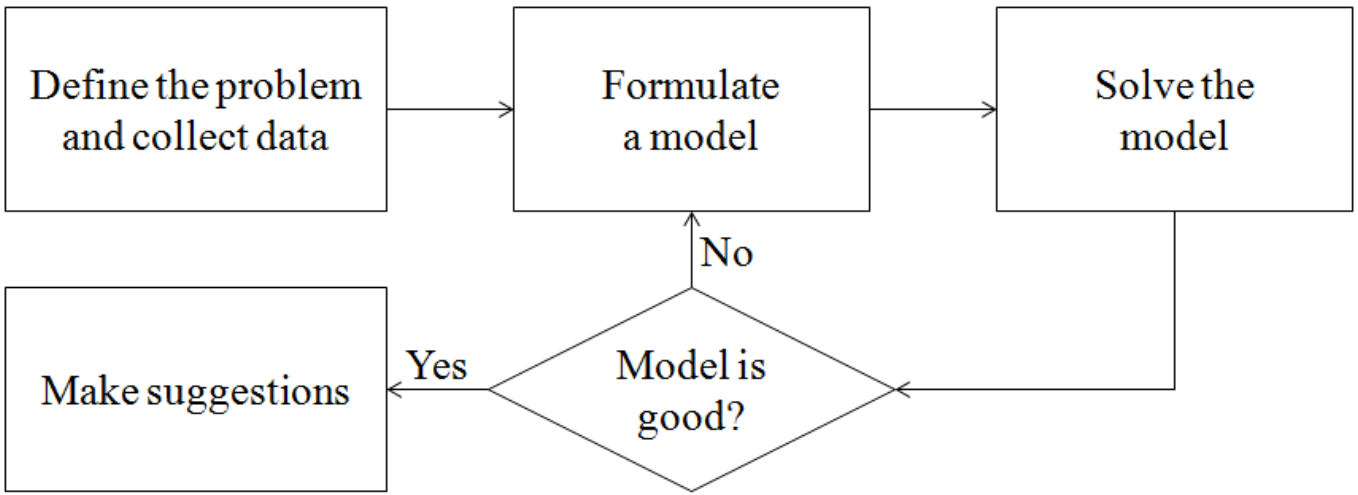
- If you are an IM student:
  - I will keep teaching this course before you graduate.
- If you are not:
  - Always welcome but think twice!
  - After this three-hour lecture, e-mail the TA **Amy Liu** to ask for a registration code.

Agenda

- Ch. 1: What is Operations Research?
- Syllabus and quiz.
- An in-class planning game.
- **Ch. 2: Introduction to modeling.**

Five steps of an OR study

- To apply OR to facilitate better decision making, we conduct **OR studies** in five steps:
  - Define the problem and collect relevant data.
  - Formulate a **mathematical model** to represent the problem.
  - Develop or apply a procedure to derive a solution from the model.
  - Test the model and refine it when needed.
  - Make managerial suggestions.



- One thing should be defined: What is a mathematical model?

Mathematical modeling

- The main "weapon" we will use in OR is **mathematical modeling**.
  - Often a mathematical model is called a **model**, a **formulation**, or a **program** in OR.
- Modeling is a way of **abstracting** a physical problem into a model with **symbols** and **formulas**.
  - Use mathematics to describe a problem.
- Why modeling?
  - We use a model to describe a problem **precisely** and **concisely**.
  - Once an **algorithm** for a type of model is developed, all problems that can be modeled in that way can be solved.

An example: step 1

- Consider the following example.
  - I have three used textbooks to sell in a second-hand market.
  - I need to bring them to the market.
  - But I may carry at most 5 kg.
  - Which book(s) should I bring?
- Step 1: Define the problem and collect relevant data.
  - The problem: To maximize the sales revenue without hurting me.
  - Data:

Book	Title	Price (NT$)	Weight (kg)
1	Calculus	500	4
2	Computer Programming	400	2
3	Operations Research	200	3

Step 2: formulating the problem

- Step 2: Precisely **formulate** (i.e., describe) the problem.
- To describe a problem:
  - **Parameters**: What cannot be controlled by us?
  - **Decision variables**: What may we control?
  - **Objective function**: What do we want?
  - **Constraints**: What are the limitations?
- Parameters:
  - 5 kg and 3 books; 500, 400, and 200 dollars; 4 kg, 2 kg, and 3 kg.
- Decision variables:
  - For each book, we may control whether to bring it. We thus define

$$x_i = \begin{cases} 1 & \text{if I carry book } i \\ 0 & \text{otherwise} \end{cases}, i = 1, ..., 3$$

as our decision variables.

Step 2: formulating the problem

- What do we want? We want to maximize the sales revenue:

$$500x_1 + 400x_2 + 200x_3.$$

- What prevent us from bringing everything? We are not strong enough:

$$4x_1 + 2x_2 + 3x_3 \leq 5.$$

- Our first model:

$$\begin{array}{rrcrcrcl} \max & 500x_1 & + & 400x_2 & + & 200x_3 & & \\ \text{s.t.} & 4x_1 & + & 2x_2 & + & 3x_3 & \leq & 5. \end{array}$$

Step 3: solving the model

- Now we want to solve the model

$$\begin{array}{rrcrcrcl} \max & 500x_1 & + & 400x_2 & + & 200x_3 & & \\ \text{s.t.} & 4x_1 & + & 2x_2 & + & 3x_3 & \leq & 5. \end{array}$$

- Wait... this problem is **unbounded**.
  - $(0, 0, 0)$ is feasible and results in \$0 as my revenue.
  - $(-1, 2, 0)$ is feasible and results in \$300 as my revenue.
  - $(-2, 4, 0)$ is feasible and results in \$600 as my revenue.
  - And so on and so on.
- We will become millionaires! What is wrong here?

Step 4: testing and revising the model

- We cannot bring "negative two" textbooks.
- How about this:

$$\begin{array}{rrcrcrcl} \max & 500x_1 & + & 400x_2 & + & 200x_3 & & \\ \text{s.t.} & 4x_1 & + & 2x_2 & + & 3x_3 & \leq & 5 \\ & x_i \geq 0 & & \forall i = 1, ..., 3. & & & & \end{array}$$

- The best solution is $(0, 2.5, 0)$. Still wrong!
- How about this:

$$\begin{array}{rrcrcrcl} \max & 500x_1 & + & 400x_2 & + & 200x_3 & & \\ \text{s.t.} & 4x_1 & + & 2x_2 & + & 3x_3 & \leq & 5 \\ & x_i \geq 0 & & \forall i = 1, ..., 3. & & & & \\ & x_i \leq 1 & & \forall i = 1, ..., 3. & & & & \end{array}$$

- The best solution is $(0.75, 1, 0)$. Still wrong!

Step 4: testing and revising the model

- What we still need: We cannot split a book:

$$x_i \in \{0, 1\} \quad \forall i = 1, ..., 3.$$

- The final formulation:[2]

$$\begin{array}{rrcrcrcl} \max & 500x_1 & + & 400x_2 & + & 200x_3 & & \\ \text{s.t.} & 4x_1 & + & 2x_2 & + & 3x_3 & \leq & 5 \\ & & & x_i \in \{0, 1\} & & \forall i = 1, ..., 3. & & \end{array}$$

- The best solution is $(0, 1, 1)$. Makes sense!

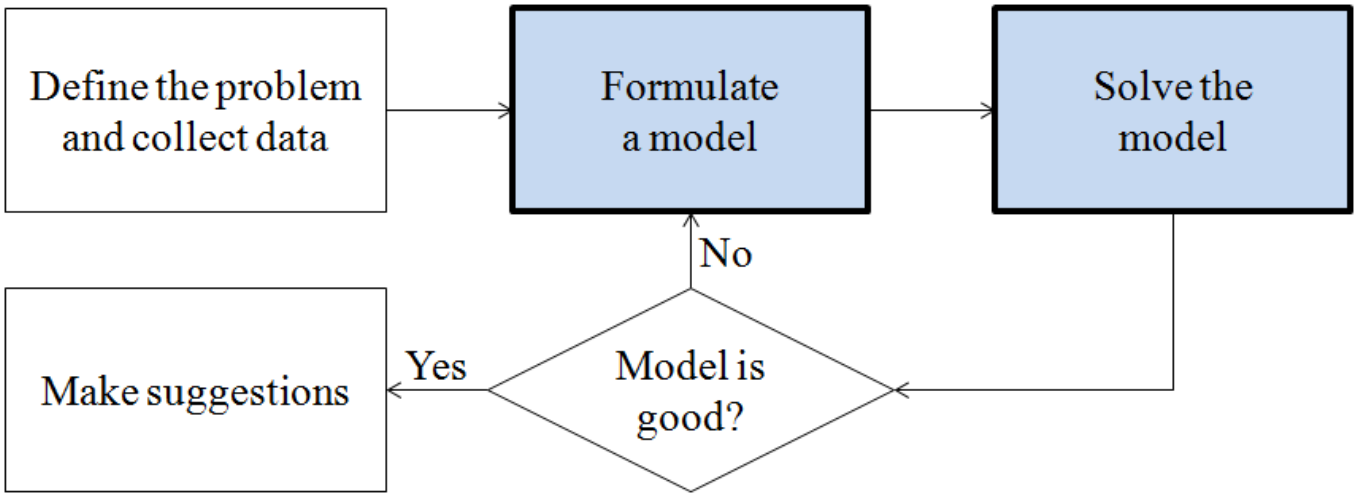
[2]The problem is an example of the **knapsack** problem, one of the most fundamental problem in Computer Science.

Lastly: Making managerial suggestions

- "$(0, 1, 1)$" means nothing to you.
  - It will also mean nothing to your boss or any manager.
  - We need **suggestions** on what to do!
  - We need to **interpret** the solution.
- Step 5: Given our model and the solution we obtain, we suggest you to sell the textbooks of Computer Programming and OR!
  - Please do so at least after you pass these courses.

Summary

- An OR study is conducted in the following five steps:



- In this course, we will focus on Steps 2 and 3.
  - These technical parts require **practices** but no **experience**.
  - You will do Step 4 by yourselves from time to time.
  - You will get a taste on Steps 1 and 5 when doing your final project.

The DFSI principle

- When you are asked to solve a decision problem in this course, you MUST do the following four things:
  - Step 1: **Define** the decision variables (and the notations you use for parameters).
  - Step 2: **Formulate** the problem as a mathematical model by writing down the objective function and constraints.
  - Step 3: **Solve** the model by finding the values for all decision variables in an optimal solution.
  - Step 4: **Interpret** the optimal solution by indicating "what to do".