Operations Research, Spring 2015
Pre-lecture Problems for Lecture 10

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Note. The deadline of submitting the pre-lecture problem is ***9:10am, May 21, 2015***. Please submit a hard copy of your work in class. Late submissions will not be accepted. Each student must submit her/his individual work. Submit ONLY the problem that counts for grades.

1. (0 point) A firm constantly sells a product to one downstream retailer. The demand rate is 1000 units per month, the unit production cost is \$10, the production rate is 3000 units per month, the setup cost is \$1000 per production run, the holding cost is \$$h$ per unit per month.

   (a) Find the EOQ as a function of h.

   (b) Is the EOQ increasing in h, decreasing in h, or neither? Mathematically prove it and intuitively explain it.

2. (0 point) A retailer sells bananas to consumers in a market. Everyday she places an order to a supplier, who will deliver the ordered bananas to the retailer at 6 am in the next morning. That is the only chance to order in a day. The unit purchasing cost is \$12 and the unit retail price is \$20. Bananas unsold until 6 pm will all be sold at a discounted price \$5 at the end of each day. The daily consumer demand follows a uniform distribution between 20 and 100.

   (a) Find the newsvendor quantity.

   (b) Suppose that the supplier now charges the retailer a \$100 shipping cost per delivery. Does that change your answer in Part (a)? Mathematically prove it and intuitively explain it.

3. (10 point) Consider the typical EOQ problem

$$\min_{q\geq 0} \ \frac{DK}{q} + \frac{qh}{2},$$

   where D is the annual demand, K is the ordering cost per order, h is the holding cost per unit per year, and q is the ordering quantity. As we discussed in the lecture videos, this is a convex program.

   (a) (3 points) Suppose that the supplier now requires the quantity of each order to be within q_L and q_H, where $q_L < q_H$. Formulate the new program and show that it is still a convex program.

   (b) (7 points) Analytically solve the new program to find the cost-minimizing order quantity. Depending on the relationship among the parameters, the optimal order quantity may be q_L, q_H, or something in between. Explicitly write down the conditions for the optimal quantity to fall in the three categories.