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Suggested Solution for Case Study 2

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1. (a) Let the parameter be

C = sets of classes (ways of taking two days off) e.g., {(1,2),(1,3),...(6,7)},
C_{con} = sets of classes (ways of taking two consecutive days off){(1,2),(2,3),...,(1,7)},
C_{sep} = sets of classes (ways of taking two separate days off){(1,3),(1,4),...,(5,7)},
C_d = sets of classes whose agents work on days $d = \{(i, j) \in C \mid i! = d,\ j! = d\}$,
D = sets of days = {1,2,...,7},
S = sets of shifts = {1,2,3},
Wf_{BD} = per week wage for a full-time agent in boarding division,
Wf_{sep} = per week compensation for a full-time agent if her/his off days are separated,
K = number of current full-time agents in boarding division,
N_{ds} = number of agents needed in shift s on day d.

Let the decision variables be

x_c = number of full-time agents in boarding division assigned to class c ,
y_{ds} = number of full-time agents in boarding division assigned in shift s on day d.

$$\begin{aligned}
\min \quad & Wf_{BD} \sum_{c \in C_{con}} x_c + (Wf_{BD} + Wf_{sep})(\sum_{c \in C_{sep}} x_c) \\
\text{s.t.} \quad & \sum_{c \in C} x_c = K \\
& \sum_{s \in S} y_{ds} = \sum_{c \in C_d} x_c \quad \forall d \in D \\
& y_{ds} \geq N_{ds} \quad \forall d \in D, s \in S \\
& x_c \geq 0 \quad \forall c \in C \\
& y_{ds} \geq 0 \quad \forall d \in D, s \in S.
\end{aligned}$$

If the model can be solved (feasible), it is possible to fulfill all demands without hiring any new agents.

(b) Let the parameter be

C = sets of classes (ways of taking two days off) e.g., {(1,2),(1,3),...(6,7)},
C_{con} = sets of classes (ways of taking two consecutive days off){(1,2),(2,3),...,(1,7)},
C_{sep} = sets of classes (ways of taking two separate days off){(1,3),(1,4),...,(5,7)},
C_d = sets of classes whose agents work on day $d = \{(i, j) \in C \mid i! = d,\ j! = d\}$,
D = sets of days = {1,2,...,7},
S = sets of shifts = {1,2,3},
Wf_{BD} = per week wage for a full-time agent in boarding division,
Wf_{sep} = per week compensation for a full-time agent if her/his off days are separated,
Wp = per week wage for a part-time agent in boarding division,
K = number of current full-time agents in boarding division,
N_{ds} = number of agents needed in shift s on day d.

Let the decision variables be

x_c = number of full-time agents in boarding division assigned to class c ,
y_{ds} = number of full-time agents in boarding division assigned in shift s on day d,
w_{ds} = number of part-time agents in boarding division assigned in shift s on day d.

$$
\begin{aligned}
\min \quad & Wf_{BD} \sum_{c \in C_{con}} x_c + (Wf_{BD} + Wf_{sep})(\sum_{c \in C_{sep}} x_c) + Wp \sum_{d \in D} \sum_{s \in S} w_{ds} \\
\text{s.t.} \quad & \sum_{c \in C} x_c \geq K \\
& \sum_{s \in S} y_{ds} = \sum_{c \in C_d} x_c \quad \forall d \in D \\
& y_{ds} + w_{ds} \geq N_{ds} \quad \forall d \in D, s \in S \\
& x_c \geq 0 \quad \forall c \in C \\
& y_{ds}, w_{ds} \geq 0 \quad \forall d \in D, s \in S.
\end{aligned}
$$

(c) Check the AMPL model file "ca02_1.c.mod" and data file "ca02_1.c.dat". After solving, the maximum profit is 67200 and we get the assignment shown in "Suggested Solution for Case Study 1".

2. (a) Let the parameter be

C = sets of classes (ways of taking two days off) e.g., {(1,2),(1,3),...(6,7)},
C_{con} = sets of classes (ways of taking two consecutive days off){(1,2),(2,3),...,(1,7)},
C_{sep} = sets of classes (ways of taking two separate days off){(1,3),(1,4),...,(5,7)},
C_d = sets of classes whose agents work on day $d = \{(i,j) \in C \mid i! = d,\ j! = d\}$,
D = sets of days = {1,2,...,7},
S = sets of shifts = {1,2,3},
G = set of groups = {IEDO, ANTS, SVVRL, KMM},
$IEDO$ = set of group = {IEDO},
$NIEDO$ = set of groups excluding IEDO = {ANTS, SVVRL, KMM},
T = set of tasks $|G| = |T|$,
T_g = set of tasks that agents in group g can do,
G_t = set of groups whose agents can do task t,
G_p = set of part-time agents can do task t,
Wf_g = per week wage for a full-time agent in group g,
Wf_{sep} = per week compensation for a full-time agent if her/his off days are separated,
Wp = per shift wage for a part time agent,
K_g = number of current full-time agents in group g,
N_{dst} = number of agents needed for task t in shift s on day d.

Let the decision variables be

x_{cg} = number of full-time agents in group g assigned to class c,
y_{dstg} = number of full-time agents in group g assigned to do task t in shift s on day d.

$$
\begin{aligned}
\min \quad & \sum_{g\in G} Wf_g \sum_{c\in C_{con}} x_{cg} + \sum_{g\in G}(Wf_g + Wf_{sep})(\sum_{c\in C_{sep}} x_{cg}) \\
\text{s.t.} \quad & \sum_{c\in C} x_{cg} = K_g \quad \forall d \in D \\
& \sum_{t\in T_g}\sum_{s\in S} y_{dstg} = \sum_{c\in C_d} x_{cg} \quad \forall d \in D, g \in G \\
& \sum_{g\in G_t} y_{dstg} \geq N_{dst} \quad \forall d \in D, s \in S, t \in NIEDO \\
& \sum_{g\in G_t} y_{dstg} \geq N_{dst} \quad \forall d \in D, s \in S, t \in IEDO \\
& x_{cg} \geq 0 \quad \forall c \in C, g \in G \\
& y_{dstg} \geq 0 \quad \forall d \in D, s \in S, t \in T, g \in G.
\end{aligned}
$$

If the model can be solved (feasible), it is possible to fulfill all demands without hiring any new agents.

(b) Let the parameter be

C = sets of classes (ways of taking two days off) e.g., {(1,2),(1,3),...(6,7)},
C_{con} = sets of classes (ways of taking two consecutive days off){(1,2),(2,3),...,(1,7)},
C_{sep} = sets of classes (ways of taking two separate days off){(1,3),(1,4),...,(5,7)},
C_d = sets of classes whose agents work on day $d = \{(i,j) \in C \mid i! = d,\ j! = d\}$,
D = sets of days = {1,2,...,7},
S = sets of shifts = {1,2,3},
G = set of groups = {IEDO, ANTS, SVVRL, KMM},
$IEDO$ = set of group = {IEDO},
$NIEDO$ = set of groups excluding IEDO = {ANTS, SVVRL, KMM},
T = set of tasks $|G| = |T|$,
T_g = set of tasks that agents in group g can do,
G_t = set of groups whose agents can do task t,
G_p = set of part-time agents can do task t,
Wf_g = per week wage for a full-time agent in group g,
Wf_{sep} = per week compensation for a full-time agent if her/his off days are separated,
Wp_t = per shift wage for a part time agent doing task t,
K_g = number of current full-time agents in group g,
N_{dst} = number of agents needed for task t in shift s on day d.

Let the decision variables be

x_{cg} = number of full-time agents in group g assigned to class c,
y_{dstg} = number of full-time agents in group g assigned to do task t in shift s on day d,
w_{dst} = number of part-time agents assigned to do task t in shift s on day d.

$$
\begin{aligned}
\min \quad & \sum_{g \in G} Wf_g \sum_{c \in C_{con}} x_{cg} + \sum_{g \in G} (Wf_g + Wf_{sep}) (\sum_{c \in C_{sep}} x_{cg}) + \sum_{t \in G_p} Wp_t \sum_{d \in D} \sum_{s \in S} w_{dst} \\
\text{s.t.} \quad & \sum_{c \in C} x_{cg} geq K_g \quad \forall d \in D \\
& \sum_{t \in T_g} \sum_{s \in S} y_{dstg} = \sum_{c \in C_d} x_{cg} \quad \forall d \in D, g \in G \\
& \sum_{g \in G_t} y_{dstg} \geq N_{dst} \quad \forall d \in D, s \in S, t \in NIEDO \\
& \sum_{g \in G_t} y_{dstg} + w_{dst} \geq N_{dst} \quad \forall d \in D, s \in S, t \in IEDO \\
& x_{cg} \geq 0 \quad \forall c \in C, g \in G \\
& y_{dstg} \geq 0 \quad \forall d \in D, s \in S, t \in T, g \in G, \\
& w_{dst} \geq 0 \quad \forall d \in D, s \in S, t \in T.
\end{aligned}
$$

(c) Check the AMPL model file "ca02_2.c.mod" and data file "ca02_2.c.dat". After solving, the maximum profit is 369600 (rounding from linear solution) and we get the assignment shown in "Suggested Solution for Case Study 1". Notice that the maximum profit for linear programming is 368750 and for integer programming is 368760.

3. (a) Let the parameter be

C = sets of classes (ways of taking two days off and working shifts on each day)

e.g., (1,2,1,1,1,1,1) means taking Monday and Tuesday off and work in the first shift from Wednesday to Sunday,

C_{con} = sets of classes (ways of taking two consecutive days off),

C_{sep} = sets of classes (ways of taking two separate days off),

C_{ds} = sets of classes whose agents work on days d in shift $s = \{(i, j, a, b, c, d, e) \in C \mid i! = d,\ j! = d\}$,

D = sets of days = {1,2,...,7},

S = sets of shifts = {1,2,3},

Wf_{BD} = per week wage for a full-time agent in boarding division,

Wf_{sep} = per week compensation for a full-time agent if her/his off days are separated,

Wp = per week wage for a part-time agent in boarding division,

K = number of full-time agents in boarding division,

N_{ds} = number of agents needed in shift s, day d.

Let the decision variables be

x_c = number of full-time agents in boarding division assigned to class c ,

y_{ds} = number of full-time agents in boarding division assigned in shift s, day d.

w_{ds} = number of part-time agents in boarding division assigned in shift s, day d.

$$
\begin{aligned}
\min \quad & Wf_{BD} \sum_{c \in C_{con}} x_c + (Wf_{BD} + Wf_{sep}) (\sum_{c \in C_{sep}} x_c) + Wp \sum_{d \in D} \sum_{s \in S} w_{ds} \\
\text{s.t.} \quad & \sum_{c \in C} x_c \geq K \\
& \sum_{s \in S} y_{ds} = \sum_{c \in C_{ds}} x_c \quad \forall d \in D \\
& y_{ds} + w_{ds} \geq N_{ds} \quad \forall d \in D, s \in S \\
& x_c \geq 0 \quad \forall c \in C \\
& y_{ds}, w_{ds} \geq 0 \quad \forall d \in D, s \in S.
\end{aligned}
$$

(b) Check the AMPL model file "ca02_3.b.mod" and data file "ca02_3.b.dat". After solving, the maximum profit is 67200'. In order to get integer solution (assignment), we set x_c and w_{ds} to be integer in our AMPL files. Notice that the objective value is the same as linear version. The assignment is as follow:

off days of full-time agents and their working schedule

off days	number of full-time agents	working shifts of other five days
12	29	11113
12	1	11123
12	19	12111
12	1	12211
12	38	13211
12	1	32111
23	1	21121
23	6	31111
34	16	13232
45	28	11121
45	64	11212
45	35	23221
45	27	22322
45	34	32132
total	**300**	

the number of agents in each shift on each day and

shift/day	1	2	3	4	5	6	7
1	2	0	0	0	13	2	0
2	27	0	1	0	5	28	0
3	0	0	22	0	20	0	0
						total	**120**

the assignment schedule

off days/ $\frac{\text{day}}{\text{shift}}$	1			2			3			4			5			6			7		
	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
12	0	0	0	0	0	0	29	0	0	29	0	0	29	0	0	29	0	0	0	0	29
12	0	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0	1	0	0	0	1
12	0	0	0	0	0	0	19	0	0	0	19	0	19	0	0	19	0	0	19	0	0
12	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	1	0	0	1	0	0
12	0	0	0	0	0	0	38	0	0	0	0	38	0	38	0	38	0	0	38	0	0
12	0	0	0	0	0	0	0	0	1	0	1	0	1	0	0	1	0	0	1	0	0
23	0	1	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	1	0	0
23	0	0	6	0	0	0	6	0	0	6	0	0	6	0	0	6	0	0	6	0	0
34	16	0	0	0	0	16	0	0	0	0	0	0	0	16	0	0	0	16	0	16	0
45	28	0	0	28	0	0	28	0	0	0	0	0	0	0	0	0	28	0	28	0	0
45	64	0	0	64	0	0	0	64	0	0	0	0	0	0	0	64	0	0	0	64	0
45	0	35	0	0	0	35	0	35	0	0	0	0	0	0	0	0	35	0	35	0	0
45	0	27	0	0	27	0	0	0	27	0	0	0	0	0	0	0	27	0	0	27	0
45	0	0	34	0	34	0	34	0	0	0	0	0	0	0	0	0	0	34	0	34	0
part-time	2	27	0	0	0	0	0	1	22	0	0	0	13	5	20	2	28	0	0	0	0
total	110	90	40	92	61	51	156	100	50	37	21	38	70	60	20	160	120	50	129	141	30