Operations Research, Spring 2017
Lecture 7: Applications of Integer Programming

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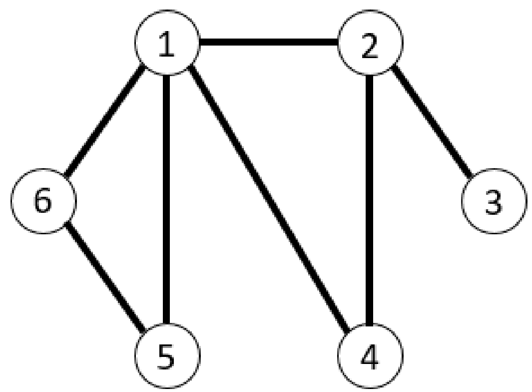
1. (2 points) There are six towns in a county. Seven roads connect these towns in the way depicted in the figure. Two towns are *adjacent* if there is a road between them. The county government wants to build hospitals in some of these towns so that each town has a hospital either in it or in an adjacent town. The number of hospitals built should be minimized.



   (a) Show that building two hospitals in towns 6 and 4 is infeasible.

   (b) Show that building two hospitals in towns 6 and 2 is optimal.

2. (5 points) There are n towns connected by m roads in a county. Two towns i and j are *adjacent* if there is a road $[i, j]$ between them. The county government wants to build hospitals in some of these towns so that each town has a hospital either in it or in an adjacent town. The number of hospitals built should be minimized. Formulate an integer program that can find an optimal plan.

   Note. This problem is called the *dominating set* problem in the field of computer science.

3. (5 points) There are m towns in a county. The county government is considering where to build at most p parks in n potential locations. The distance between town i and location j is d_{ij}. The population at town i is h_i. The government wants to minimize the *average* distances for each people to move to her/his closest park. Formulate an integer program that achieves this goal.

   Note. This problem is called the *p-median* problem in the subject of facility location problems.

4. (5 points) There are m towns in a county. The county government is considering where to build at most p parks in n potential locations. The distance between town i and location j is d_{ij}. The population at town i is h_i. The government wants to minimize the *maximum* distances for each people to move to her/his closest park. Formulate an integer program that achieves this goal.

   Note. This problem is called the *p-center* problem in the subject of facility location problems.

5. n jobs must be scheduled on one single machine. For job i, the required processing time is p_i. Moreover, there is a due time d_i and a release time r_i. The machine may start to process job i only after r_i.

   (a) (3 points) Formulate an integer program to determine whether it is possible to schedule all jobs with no tardy job.

   (b) (3 points) Formulate an integer program to minimize the number of tardy jobs.

6. (5 points) n jobs must be scheduled on m parallel machines. All machines are identical. For job j, the required processing time is p_j, and the amount of benefit that can be collected upon completing the job is b_j. A job must be processed by exactly one machine. For each machine, let its *total processing time* and *total benefit* be the sums of processing times and benefits of all jobs assigned to it, respectively. The capacity of each machine is K, which is the maximum total processing time that a machine may have. Formulate an integer program to maximize the minimum benefit earned by a machine.