Operations Research, Spring 2014
Homework 2

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Note. The deadline of this homework is ***1pm, March 6, 2014***. Please put a hard copy of the work into the instructor's mailbox on the first floor of the Management Building II by the due time. Late submissions will not be accepted. Each student must submit her/his individual work.

1. (10 points; 5 points each) Answer the following questions regarding extreme points.
   (a) For the set $\{x \in \mathbb{R}^n | x_i \geq 0 \quad \forall i = 1, ..., n\}$, show that $(0, ..., 0, 1)$ is not an extreme point.
   (b) The the set $\mathbb{N} \cup \{0\} = \{0, 1, 2, ...\}$, find all the extreme points.

2. (10 points) Graphically solve the following LP:

$$\begin{array}{rl} \min & 2x_1 - x_2 \\ \text{s.t.} & x_1 + x_2 \leq 6 \\ & x_1 - x_2 \geq 0 \\ & 2x_1 + x_2 \geq 6 \\ & x_1, x_2 \geq 0. \end{array}$$

3. (0 points; this is an example for LP formulation) Each day, workers at the Gotham City Police Department work two 6-hour shifts chosen from 12 A.M. to 6 A.M., 6 A.M. to 12 P.M., 12 P.M. to 6 P.M., and 6 P.M. to 12 A.M. The following number of workers are needed during each shift: 12 A.M. to 6 A.M. – 15 workers; 6 A.M. to 12 P.M. – 5 workers; 12 P.M. to 6 P.M. – 12 workers; 6 P.M. to 12 A.M. – 6 workers. Workers whose two shifts are consecutive are paid \$12 per hour; workers whose shifts are not consecutive are paid \$18 per hour. Formulate an LP that can be used to minimize the cost of meeting the daily workforce demands of the Gotham City Police Department.

   Suggested solution. To formulate this problem, we label the four shifts as in the following table.

Shift Number	1	2	3	4
Time	0 – 6	6 – 12	12 – 18	18 – 24

   The decision variables are

$$x_{ij} = \text{number of officers working at shifts } i \text{ and } j,\ i = 1, ..., 4,\ j = i + 1, ..., 4.$$

   The problem can then be formulated as

$$\begin{array}{rll} \min & 144(x_{12} + x_{23} + x_{34} + x_{14}) + 216(x_{13} + x_{24}) & \text{(Total wage)} \\ \text{s.t.} & x_{12} + x_{13} + x_{14} \geq 15 & \text{(Demand in shift 1)} \\ & x_{12} + x_{23} + x_{24} \geq 5 & \text{(Demand in shift 2)} \\ & x_{13} + x_{23} + x_{34} \geq 12 & \text{(Demand in shift 3)} \\ & x_{14} + x_{24} + x_{34} \geq 6 & \text{(Demand in shift 4)} \\ & x_{ij} \geq 0 \quad \forall\ i = 1, ..., 4, j = i + 1, ..., 4. & \end{array}$$

   Here $144 = 12 \times 12$ is the per worker wage for one who works in two consecutive shifts. Similarly, $216 = 18 \times 12$ is that for one who works in two nonconsecutive shifts.[1]

[1] It does not matter if you use 12 and 18 instead of 144 and 216 in the formulation. The optimal solution will not be affected. Using 144 and 216 is suggested, however, because by doing so the objective function gives the amount of total payments as we desire.

4. (10 points) The IM City Police Department employs 30 police officers. Each officer works 5 days per week. The crime rate fluctuates with the day of the week, so the number of police officers required each day depends on which day of the week it is: Saturday, 25; Sunday, 16; Monday, 12; Tuesday, 20; Wednesday, 18; Thursday, 22; Friday, 25. The police department wants to schedule police officers to minimize the number whose days off are NOT consecutive. Formulate an LP that will accomplish this goal.

   Note. For formulation problems, please try to make your formulation as clear and precise as possible. Reading the examples on lecture slides and the textbook definitely helps.

5. (10 points) John produces the drug MakeMeStrong from four chemicals. Today he must produce EXACTLY 1,000 lb of the drug. The three active ingredients in MakeMeStrong are A, B, and C. By weight, at least 5% of MakeMeStrong must consist of A, at least 4% of B, and at least 2% of C. The cost per pound of each chemical and the amount of each active ingredient in 1 lb of each chemical are given in the following table. It is necessary that at least 100 lb of chemical 2 be used. Formulate an LP whose solution would determine the cheapest way of producing today's batch of MakeMeStrong.

Chemical	Cost ($ per lb)	A	B	C
1	8	.04	.02	.01
2	12	.06	.05	.01
3	13	.10	.03	.03
4	15	.11	.09	.04

6. (10 points) Mary uses chemicals 1, 2, ..., n to produce drugs 1, 2, ..., m. Drug i must be at least M_{ij}% chemical j. Up to D_i oz of drug i can be sold at $\$P_i$ per oz. Up to S_j oz of chemical j can be purchased at $\$C_j$ per oz. Formulate an LP that can be used to maximize Mary's profits.

7. (10 points) IM Oil has refineries in Kaohsiung and Taipei. Currently, the Kaohsiung refinery can refine up to 2 million barrels of oil per year, and the Taipei refinery up to 3 million. Once refined, oil is shipped to two distribution points: Hsinchu and Taichung. IM Oil estimates that each distribution point can sell up to 5 million barrels per year. Because of differences in shipping and refining costs, the profit earned (in US dollars) per million barrels of oil shipped depends on where the oil was refined and on the point of distribution (see the table below). At this moment, IM Oil is considering expanding the capacity of each refinery. Each million barrels of annual refining capacity that is added will cost $120,000 for the Kaohsiung refinery and $150,000 for the Taipei refinery. Capacity can only be added now but can be used in the future ten years. Formulate an LP that maximizes IM's profits less expansion costs over a ten-year period.

From	To Hsinchu	To Taichung
Kaohsiung	$16,000	$19,000
Taipei	$22,000	$18,000

8. (40 points) In this problem, we would like to invite you to design problems for your classmates! Please create a scenario in which one has a decision to make and the decision making can be supported by Linear Programming. Describe the situation clearly and formulate an LP that solves the decision making problem. There is no restriction on the situation: As long as the decision can be made based on the outcome of an LP, it is fine. You may write either in English or Chinese. If you write in English and your problem is easy to understand, you will get some bonus points. The expected complexity of your problem is that most of your classmates may formulate a correct LP in 15 to 20 minutes. It is not good if it is too easy or too hard. The breakdown of grades is: (1) 25 points for the preciseness of your problem description and correctness of your solution, (2) 10 points for the complexity, (3) 5 points for how interesting your problem is, and (4) 5 points for writing easy-to-read English. Some best problems will be selected to become lecture or homework problems (subject to your approval).