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Suggested Solution for Homework 7

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1. (a) The shipping cost is \$300 with quantity less or equal to 500 kilograms. The optimal order quantity will be less or equal to 500 kilograms, otherwise we can split the order and get lower total cost. For xample, the order quantity 600 kilograms can be splited to 300, 300 kilograms, which has same shipping cost but lower holding cost. Then we can formulate the problem

$$\begin{aligned} \min \quad & 300\lceil \frac{12000}{q} \rceil + 15q \\ \text{s.t.} \quad & q \leq 500 \\ & q \geq 0. \end{aligned}$$

By using Excel we get two optimal solution which are 500 kilograms and 480 kilograms.

(bcd) The information with order quantity 500, 480 kilograms are shown below

Order Quantity	500 kilograms	480 kilograms
Order times	24	25
Cycle time	15 days	14.4 days
Shipping cost	7200	7500
Holding cost	7500	7200
Total cost	14700	14700
Recorder point	$\frac{500}{3}$ kilograms	$\frac{560}{3}$ kilograms

2. The ordering cost, holding cost, and total cost as shown following

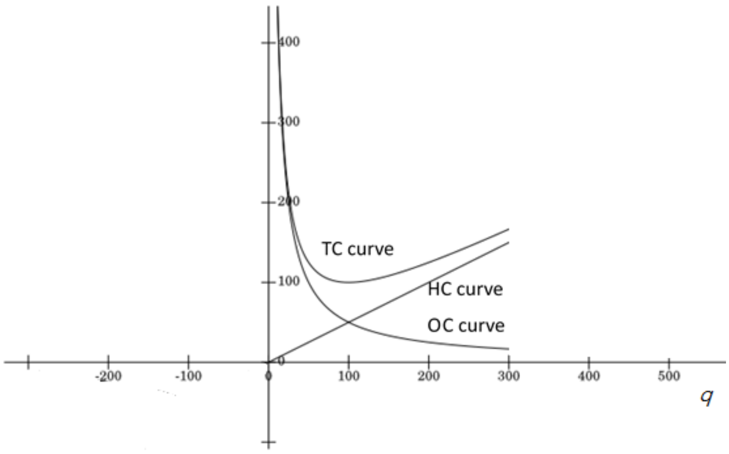

Figure 1: Ordering cost, Holding cost, Total cost for Problem 2

3. (a) $q^* = \sqrt{\frac{2KD}{h(1-\frac{D}{r})}}, q' = \sqrt{\frac{8KD}{h(1-\frac{2D}{r})}}$

$\frac{q'}{q^*} = 2\sqrt{\frac{r-D}{r-2D}}$

(b) $TC(q) = \frac{KD}{q} + \frac{hq(1-\frac{D}{r})}{2}$

$\frac{TC(q')}{TC(q^*)} = \sqrt{\frac{r-D}{r-2D}} + \frac{1}{4}\sqrt{\frac{r-2D}{r-D}}$

(c) Yes, the EPQ model is robust. Let $S = \sqrt{\frac{r-D}{r-2D}}$. When $q^{'}$ is $2S$ times of q^*, the total cost become $\frac{1}{4S}+1$ times than optimal value. Since $S \geq 1$, order quantity has small influence to the total cost.

4. ($\Rightarrow$) When ordering cost is equal to holding cost, $\frac{KD}{q} = \frac{hq}{2}$. Then $q^2 = \frac{2KD}{h}, q = \sqrt{\frac{2KD}{h}}$ equal to q^*. Therefore when ordering cost is equal to holding cost, the order quantity is optimal order quantity.

($\Leftarrow$) When order quantity is optimal order quantity, $q = q^* = \sqrt{\frac{2KD}{h}}$. Ordering cost : $OC(q) = \frac{KD}{q} = \sqrt{\frac{hKD}{2}}$. Holding cost : $HC(q) = \frac{hq}{2} = \sqrt{\frac{hKD}{2}} = OC(q)$. Therefore when order quantity is optimal order quantity, the ordering cost is equal to holding cost.

5. (a) The overage cost $c_o = \$10$, the underage cost $c_u = \$30$.

$$\begin{aligned} 1 - F(q^*) &= \frac{c_o}{c_o + c_u} \\ \Rightarrow \quad \frac{200 - q^*}{200} &= \frac{1}{4} \\ \Rightarrow \quad q^* &= 150 \end{aligned}$$

the optimal order quantity is 150.

(b) Because

$$\begin{aligned} 1 - F(q^*) &= \frac{c_o}{c_o + c_u} \\ \Rightarrow \quad \text{Normal}(\frac{q^* - 150}{30}) &= \frac{3}{4} \\ \Rightarrow \quad q^* &= 170.235, \end{aligned}$$

the optimal order quantity is 170.235.

(c) Because

$$\begin{aligned} 1 - F(q^*) &= \frac{c_o}{c_o + c_u} \\ \Rightarrow \quad e^{-0.02q^*} &= \frac{1}{4} \\ \Rightarrow \quad q^* &= 69.314, \end{aligned}$$

the optimal order quantity is 69.314.

6. The overage cost $c_o = \$12$, the underage cost $c_u = \$30$.

$$\begin{aligned} 1 - F(q^*) &= \frac{c_o}{c_o + c_u} \\ \Rightarrow \quad \frac{200 - q^*}{200} &= \frac{12}{42} \\ \Rightarrow \quad q^* &= \frac{1000}{7} \end{aligned}$$

Because there are lot size constraints, we should compare order quantities 140 and 150. The expected total cost function for this problem is

$$TC(q) = \frac{1}{200}\left(\frac{1}{2}(c_o + c_u)q^2 + \frac{1}{2}(200)^2 c_u - 200 c_u q\right).$$

If the order quantity is 140, the total expected costs increases by \$0.76. If the order quantity is 150, the total expected costs increases by \$4.7. Therefore, we should choose 140 to be our order quantity.

7. omitted