Operations Research Lab Session

TA: 陳嘉豪 (Jack Chen) 陳宗霆 (Tim Chen) 2014/03/24



1. Homework 4 illustration

Problem 3

Non-basic variable

| <i>c</i> ₁ | <i>c</i> ₂ | 0 | 0 | 0 | 0 | 10 |
|-----------------------|-----------------------|---|---|-------|---|----|
| 4 | a_1 | 1 | 0 | a_2 | 0 | b |
| -1 | -5 | 0 | 1 | -1 | 0 | 2 |
| a_3 | -3 | 0 | 0 | -4 | 1 | 3 |
| | | | | | | I |

Basic variable

Problem 3(a)

| <i>c</i> ₁ | <i>c</i> ₂ | 0 | 0 | 0 | 0 | 10 |
|-----------------------|-----------------------|---|---|-------|---|----|
| 4 | a_1 | 1 | 0 | a_2 | 0 | b |
| -1 | -5 | 0 | 1 | -1 | 0 | 2 |
| a_3 | -3 | 0 | 0 | -4 | 1 | 3 |

tableau optimal => (1) find a solution (2) find multiple solutions $c_1 \ge 0, c_2 \ge 0$

Problem 3(b)

| <i>c</i> ₁ | <i>C</i> ₂ | 0 | 0 | 0 | 0 | 10 |
|-----------------------|-----------------------|---|---|-------|---|----|
| 4 | a_1 | 1 | 0 | a_2 | 0 | b |
| -1 | -5 | 0 | 1 | -1 | 0 | 2 |
| a_3 | -3 | 0 | 0 | -4 | 1 | 3 |

unbounded => has entering variable, has no leaving variable

$$c_2 < 0 \qquad \qquad a_1 \le 0$$

Problem 3(c)

10 C_1 C_{2} 1 b 4 a_1 a_{2} 0 1 -5 -1 3 0 -4 -3 a_3 current bfs degenerate => RHS has 0 RHS has $0 \Rightarrow m+1$ variable value are $0 \Rightarrow m$ bfs in same point

(0, 0, 0, 2, 0, 3) choose 3 zero as non-basis variable

Problem 3(c)

LP is degenerated



Same ratio of leaving variable

Problem 3(d)



Enter
$$x_1$$
 and leave $x_6 \quad \frac{b}{4} \ge \frac{3}{a_3}$

Problem 3(e) modified

| <i>c</i> ₁ | <i>C</i> ₂ | 0 | 0 | 0 | 10 |
|-----------------------|-----------------------|---|---|---|----|
| 4 | a_1 | 1 | 0 | 0 | b |
| -1 | -5 | 0 | 1 | 0 | 2 |
| a_3 | -3 | 0 | 0 | 1 | 3 |

b > 0, multiple solutions

solution => c_1 , c_2 non-negative $c_1 \ge 0, c_2 \ge 0$ multiple solutions => c_1 , c_2 at least one is 0 $c_1c_2 = 0$

Problem 3(e)

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| <i>c</i> ₁ | c_2 | 0 | 0 | 0 | 0 | 10 |
|-----------------------|-------|---|---|-------|---|----|
| 4 | a_1 | 1 | 0 | a_2 | 0 | b |
| -1 | -5 | 0 | 1 | -1 | 0 | 2 |
| a_3 | -3 | 0 | 0 | -4 | 1 | 3 |

 $c_1 \geq 0, c_2 \geq 0$

Problem 3(f) modified

| <i>c</i> ₁ | <i>C</i> ₂ | 0 | 0 | 0 | 10 |
|-----------------------|-----------------------|---|---|---|----|
| 4 | a_1 | 1 | 0 | 0 | b |
| -1 | -5 | 0 | 1 | 0 | 2 |
| a_3 | -3 | 0 | 0 | 1 | 3 |

b = 0, multiple solutions When $c_1 = 0$ (degenerate has same solution!) => $c_1 \ge 0$ When $c_2 = 0$, $a_1 > 0$ (degenerate) => $c_2 = 0$, $a_1 \le 0$

Problem 3(f)

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| c_1 | <i>c</i> ₂ | 0 | 0 | 0 | 0 | 10 |
|-------|-----------------------|---|---|-------|---|----|
| 4 | a_1 | 1 | 0 | a_2 | 0 | b |
| -1 | -5 | 0 | 1 | -1 | 0 | 2 |
| a_3 | -3 | 0 | 0 | -4 | 1 | 3 |

 $a_2 \le 0, c_1 \ge 0, c_2 \ge 0 \implies$ multiple solutions