Operations Research

Lab Session

TA: 陳嘉豪 (Jack Chen)
陳宗霆 (Tim Chen)
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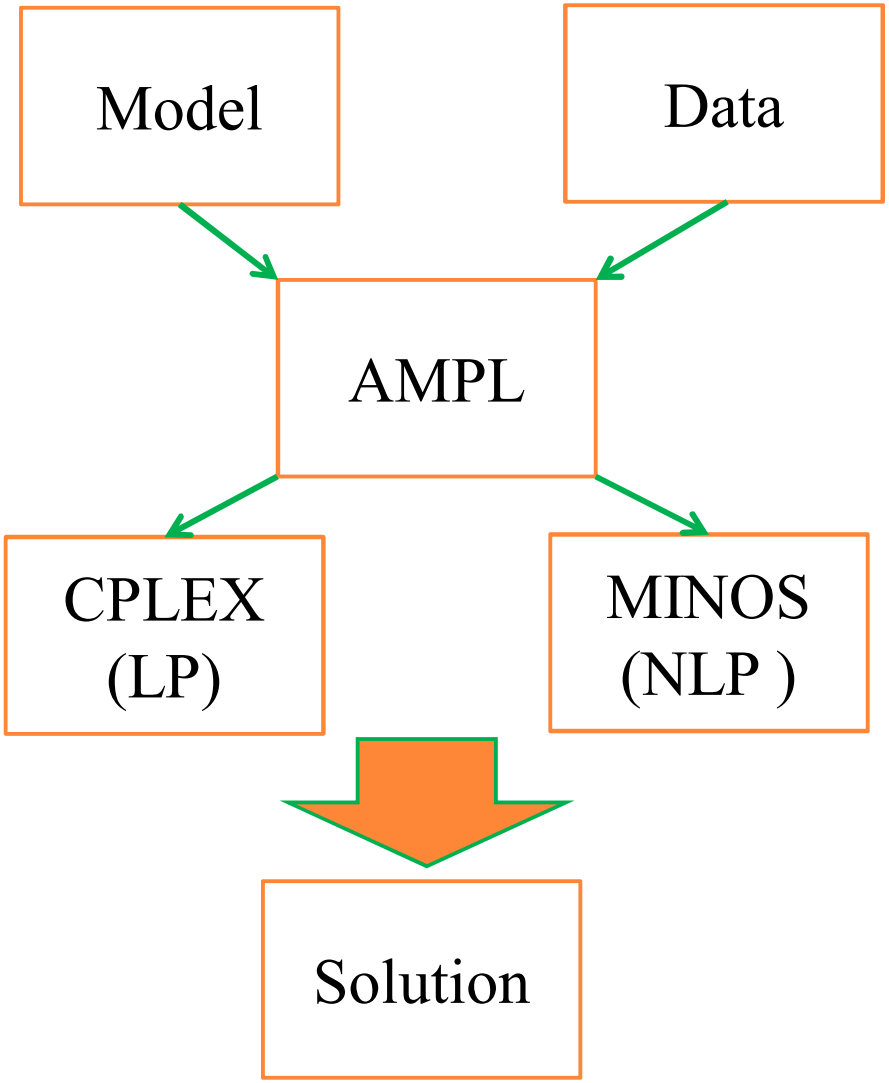
Outline

1. AMPL (A Mathematical Programming Language)

AMPL



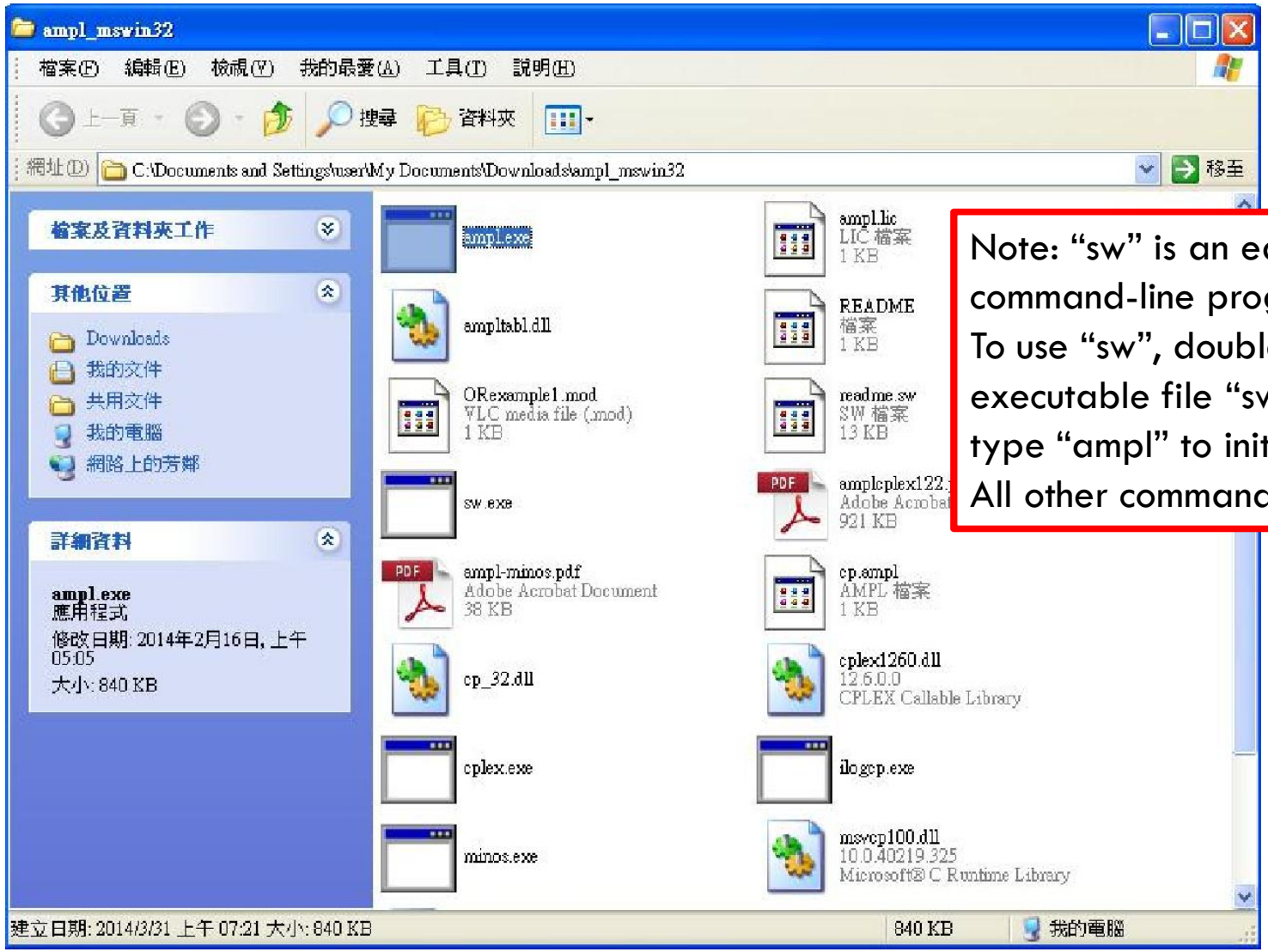
Solver

Transform method

$$\begin{aligned}\text{Max} \quad & 2x_1 + 3x_2 \\ \text{s.t.} \quad & 5x_1 + 7x_2 \le 48 \\ & 4x_1 + 2x_2 \le 20 \\ & x_i \ge 0 \quad \forall\, i = 1,2.\end{aligned}$$

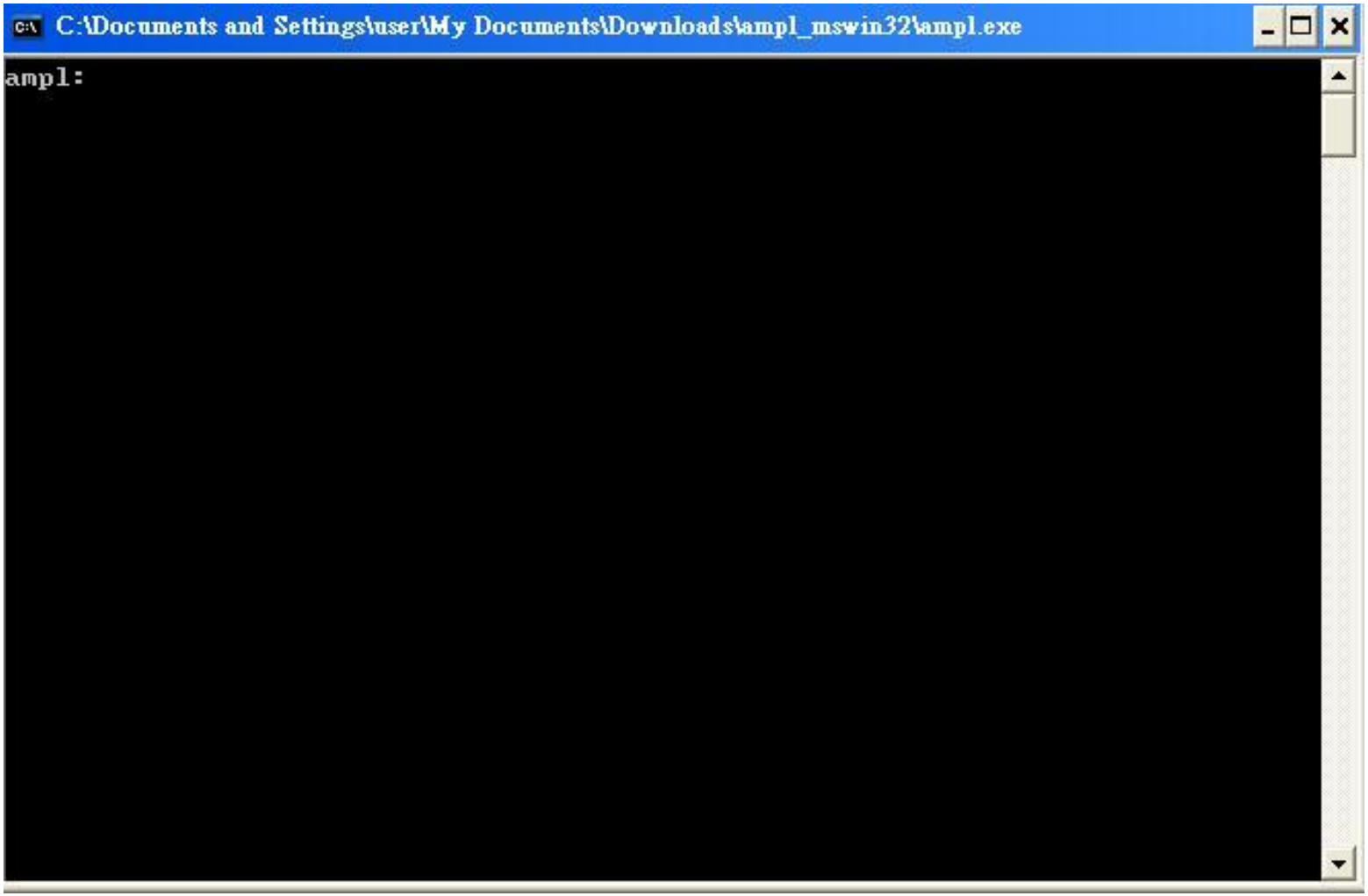
```
var x1; #define variable
var x2;

maximize Z: 2*x1 + 3*x2; # objected function
subject to constraintA: 5*x1 + 7*x2 <= 48; #constraint
subject to constraintB: 4*x1 + 7*x2 <= 20;
subject to constraintC:    x1 >= 0;
subject to constraintD:    x2 >= 0;
```

(ORexample1.mod)

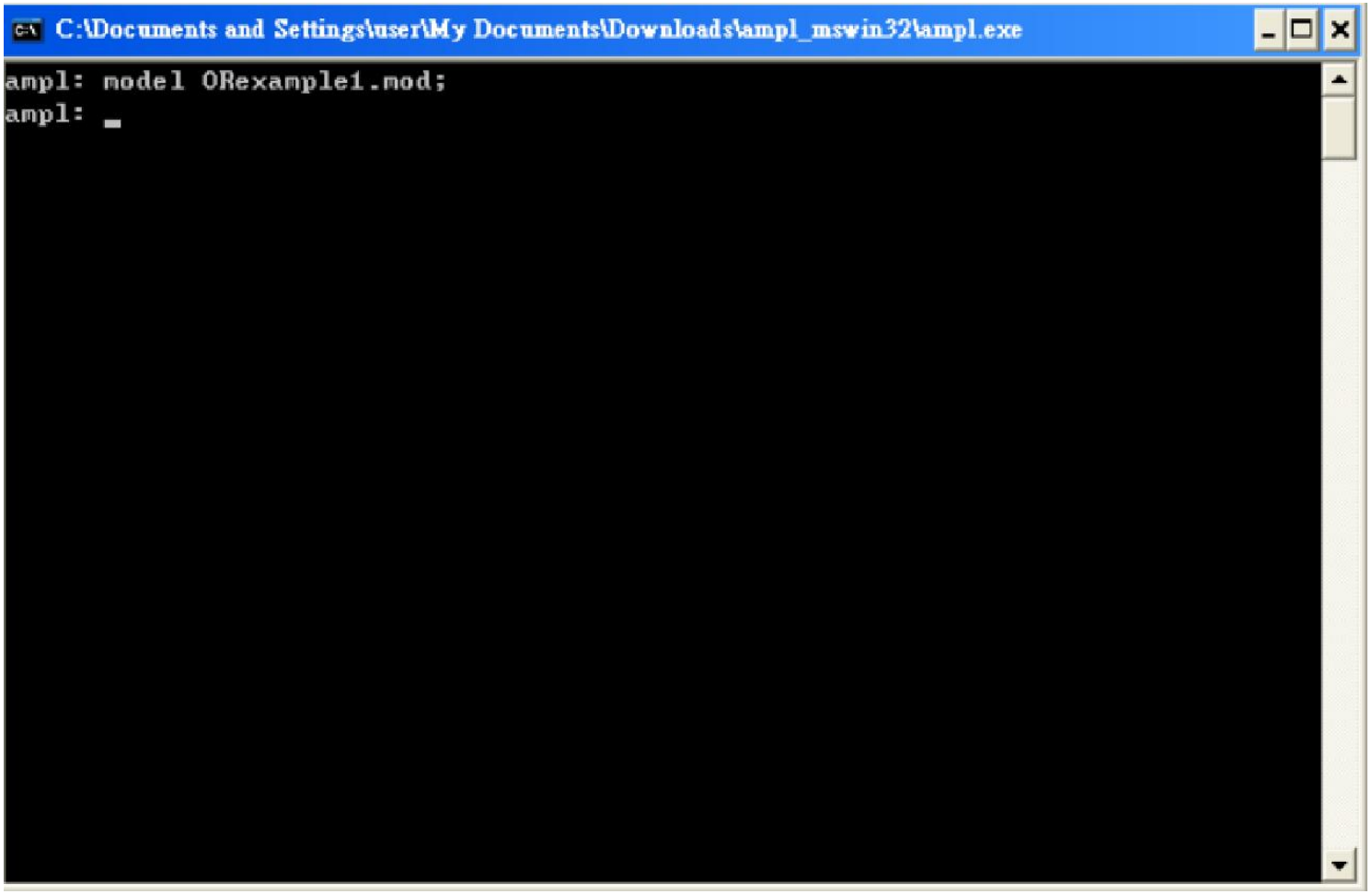


Note: “sw” is an easier-to-use command-line program
To use “sw”, double click the executable file “sw” and then type “ampl” to initiate “ampl”
All other commands are the same

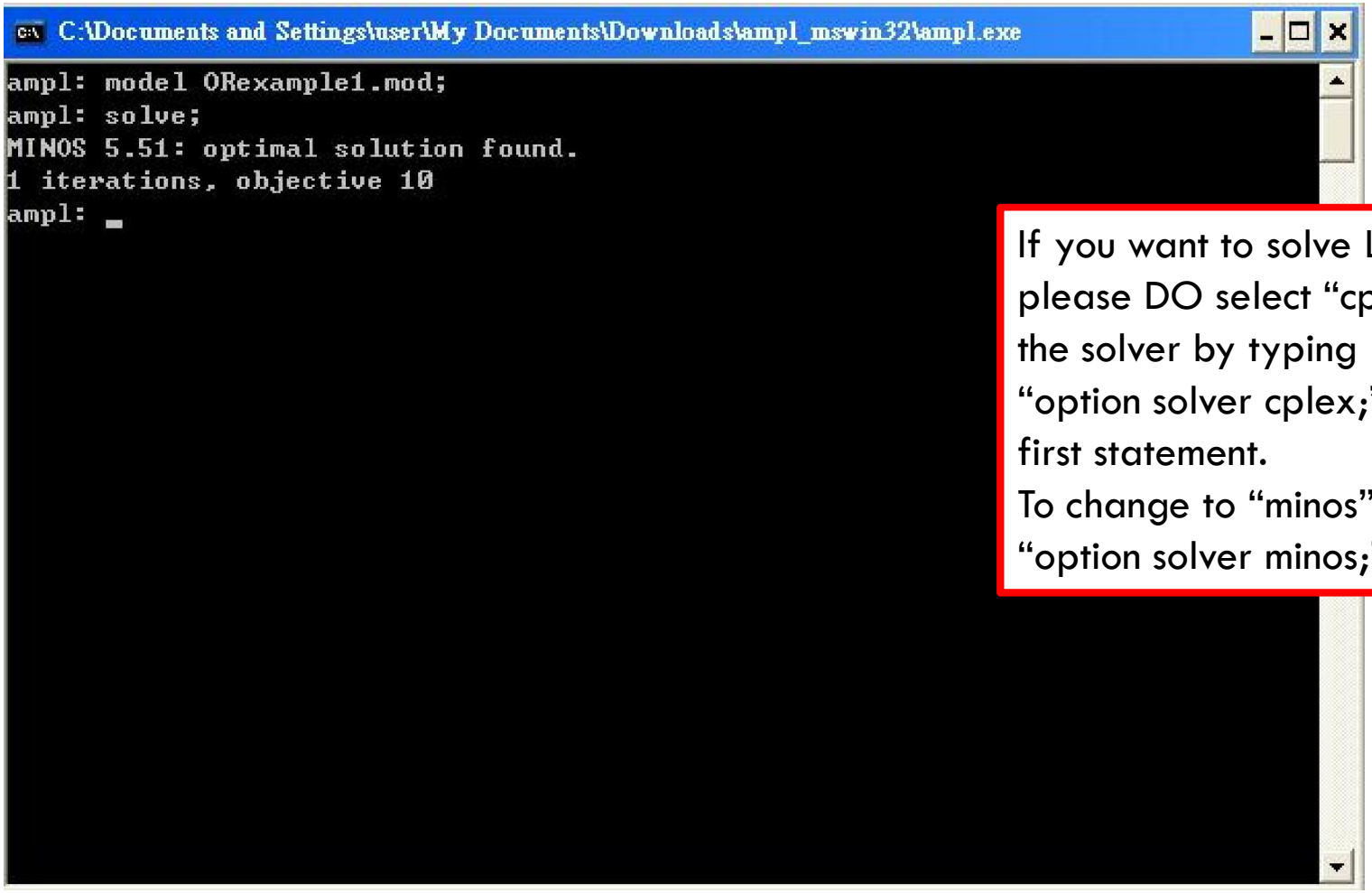
interface


C:\Documents and Settings\user\My Documents\Downloads\ampl_mswin32\ampl.exe
ampl:

Input file


C:\Documents and Settings\user\My Documents\Downloads\ampl_mswin32\ampl.exe
ampl: model ORexample1.mod;
ampl:

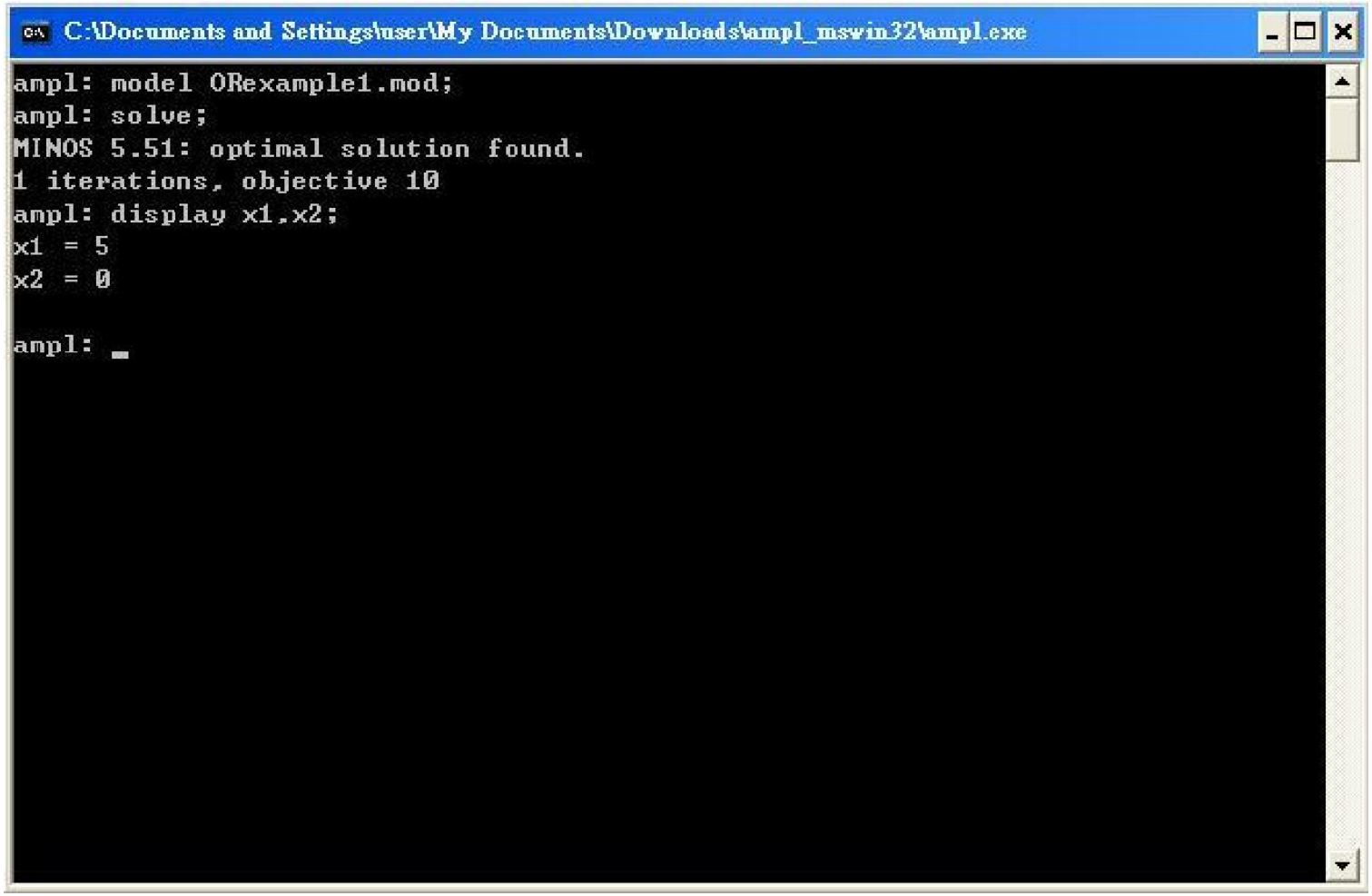
Solve operation



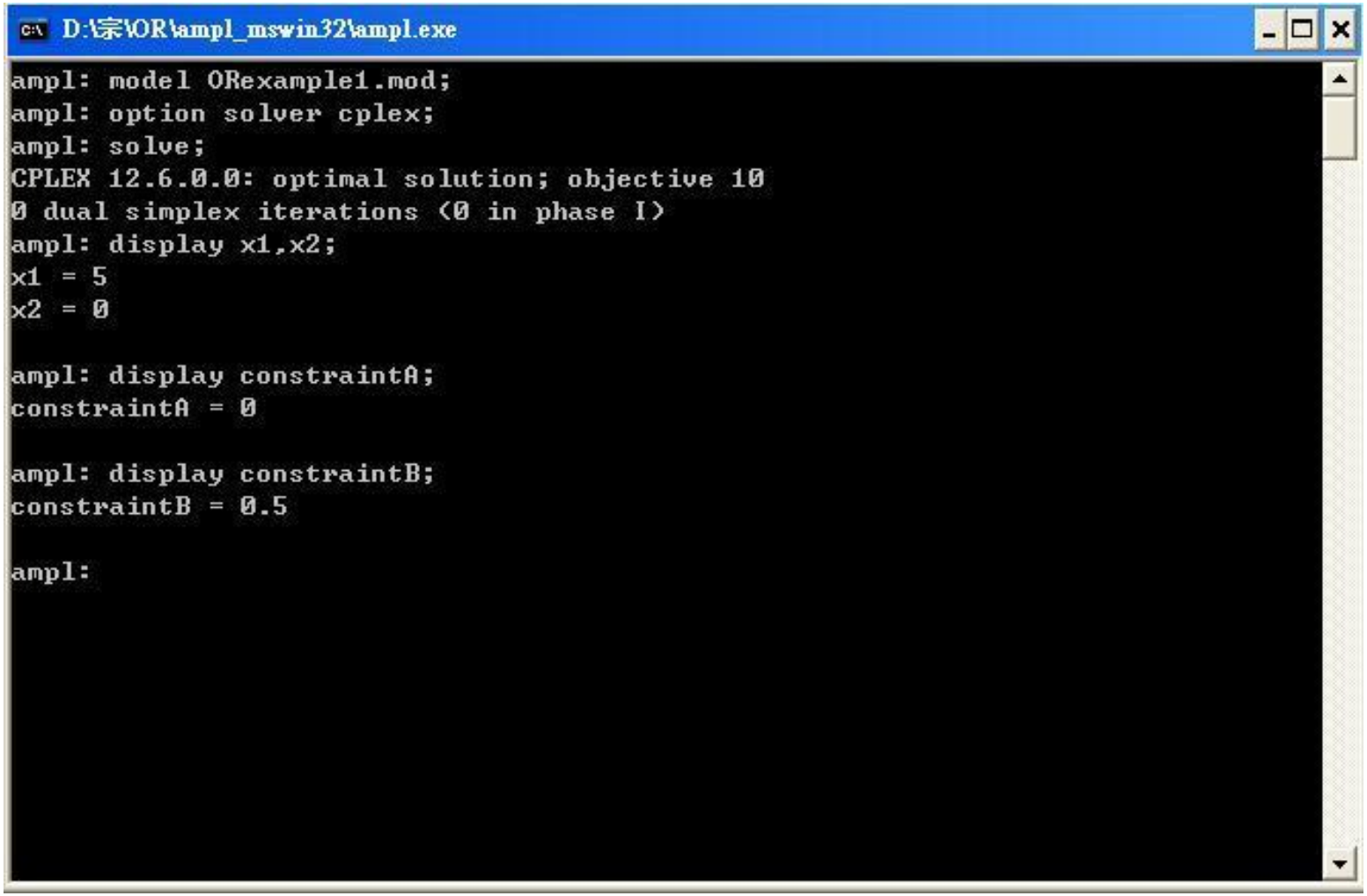
If you want to solve LPs or IPs, please DO select "cplex" as the solver by typing "option solver cplex;" as the first statement.
To change to "minos", type "option solver minos;"

option solver cplex; (assign solver)

Display operation



Shadow price



Reset

- If you encounter some compilation errors, use the statement “reset;” (maybe multiple times) to clean program items loaded in memory.
  - You must do this before you reload model and data files!

More difficult problem

5. (10 points) John produces the drug MakeMeStrong from four chemicals. Today he must produce EXACTLY 1,000 lb of the drug. The three active ingredients in MakeMeStrong are A, B, and C. By weight, at least 5% of MakeMeStrong must consist of A, at least 4% of B, and at least 2% of C. The cost per pound of each chemical and the amount of each active ingredient in 1 lb of each chemical are given in the following table. It is necessary that at least 100 lb of chemical 2 be used. Formulate an LP whose solution would determine the cheapest way of producing today's batch of MakeMeStrong.

Chemical	Cost ($ per lb)	A	B	C
1	8	.04	.02	.01
2	12	.06	.05	.01
3	13	.10	.03	.03
4	15	.11	.09	.04

More difficult problem

Let the decision variables be

$$x_i = \text{pounds of chemical } i \text{ used}, \; i = 1, \ldots, 4.$$

The complete formulation of this problem is

$$
\begin{array}{lrcrcrcrcll}
\min & 8x_1 & + & 12x_2 & + & 13x_3 & + & 15x_4 & & & \\
\text{s.t.} & x_1 & + & x_2 & + & x_3 & + & x_4 & = & 1000 & \text{(Total amount produced)} \\
 & 0.04x_1 & + & 0.06x_2 & + & 0.1x_3 & + & 0.11x_4 & > & 50 & \text{(Quality: ingredient A)} \\
 & 0.02x_1 & + & 0.05x_2 & + & 0.03x_3 & + & 0.09x_4 & > & 40 & \text{(Quality: ingredient B)} \\
 & 0.01x_1 & + & 0.01x_2 & + & 0.03x_3 & + & 0.04x_4 & > & 20 & \text{(Quality: ingredient C)} \\
 & & & x_2 & & & & & > & 100 & \text{(Least amount of chemical 2)} \\
 & & & & & & & x_i & > & 0 & \forall\, i = 1, \ldots, 4.
\end{array}
$$

We have some parameters and sets!!!

- set => set
- parameter => param
- variable => var

Chemical	Cost ($ per lb)	A	B	C
1	8	.04	.02	.01
2	12	.06	.05	.01
3	13	.10	.03	.03
4	15	.11	.09	.04

```
set I :=  1..4; # number of chemical
set J := A B C; #number of ingredient

param Ingredient :
          A       B       C       :=
1         0.04    0.02    0.01
2         0.06    0.05    0.01
3         0.1     0.03    0.03
4         0.11    0.09    0.4;
```

Grammar:

var [name] {index1,index2,...} {attributes} ;

屬性

Example:

```
var x1;
var x2 integer; #x2 is integer
var x3 := 1;  #x3 initial value is 1
var x4{i in I}; #x4 is array
var x5 >= 0;  #x5 is nonnegative
var x6 binary; #x6 is binary
```

Grammar:
param [name] {index1,index2,...} {attributes};

Grammar:
set [set name] [set expression];

Example:
set I := 1..4;
set J := A B C;

Let the decision variables be

$$x_i = \text{pounds of chemical } i \text{ used}, \ i = 1, \ldots, 4.$$

The complete formulation of this problem is

$$\begin{array}{lrcrcrcrcll}
\min & 8x_1 & + & 12x_2 & + & 13x_3 & + & 15x_4 & & & \\
\text{s.t.} & x_1 & + & x_2 & + & x_3 & + & x_4 & = & 1000 & \text{(Total amount produced)} \\
& 0.04x_1 & + & 0.06x_2 & + & 0.1x_3 & + & 0.11x_4 & > & 50 & \text{(Quality: ingredient A)} \\
& 0.02x_1 & + & 0.05x_2 & + & 0.03x_3 & + & 0.09x_4 & > & 40 & \text{(Quality: ingredient B)} \\
& 0.01x_1 & + & 0.01x_2 & + & 0.03x_3 & + & 0.04x_4 & > & 20 & \text{(Quality: ingredient C)} \\
& & & x_2 & & & & & > & 100 & \text{(Least amount of chemical 2)} \\
& & & & & & & x_i & > & 0 & \forall\, i = 1, \ldots, 4.
\end{array}$$

```
var x{i in I}; # pound of chemical

minimize Z: sum{i in I} Cost[i] * x[i];
subject to Total:  sum{i in I} x[i] =1000;  #x1+x2+x3+x4 = 1000;
subject to Quality{j in J}: sum{i in I} Ingredient[i,j]*x[i] >= Lower[j];
subject to Amount1: x[1] >= 0;
subject to Amount2: x[2] >= 100;
subject to Amount3 : x[3] >= 0;
subject to Amount4 : x[4] >= 0;
```

Let the decision variables be

$$x_i = \text{pounds of chemical } i \text{ used, } i = 1, \ldots, 4.$$

The complete formulation of this problem is

$$\begin{array}{lrcrcrcrcll}
\min & 8x_1 & + & 12x_2 & + & 13x_3 & + & 15x_4 & & & \\
\text{s.t.} & x_1 & + & x_2 & + & x_3 & + & x_4 & = & 1000 & \text{(Total amount produced)} \\
& 0.04x_1 & + & 0.06x_2 & + & 0.1x_3 & + & 0.11x_4 & > & 50 & \text{(Quality: ingredient A)} \\
& 0.02x_1 & + & 0.05x_2 & + & 0.03x_3 & + & 0.09x_4 & > & 40 & \text{(Quality: ingredient B)} \\
& 0.01x_1 & + & 0.01x_2 & + & 0.03x_3 & + & 0.04x_4 & > & 20 & \text{(Quality: ingredient C)} \\
& & & x_2 & & & & & > & 100 & \text{(Least amount of chemical 2)} \\
& & & & & & & x_i & > & 0 & \forall\, i = 1, \ldots, 4.
\end{array}$$

subject to Amount1: x1 >= 0;

subject to Amount2: x2 >= 100;

subject to Amount3 : x3 >= 0;

subject to Amount4 : x4 >= 0;

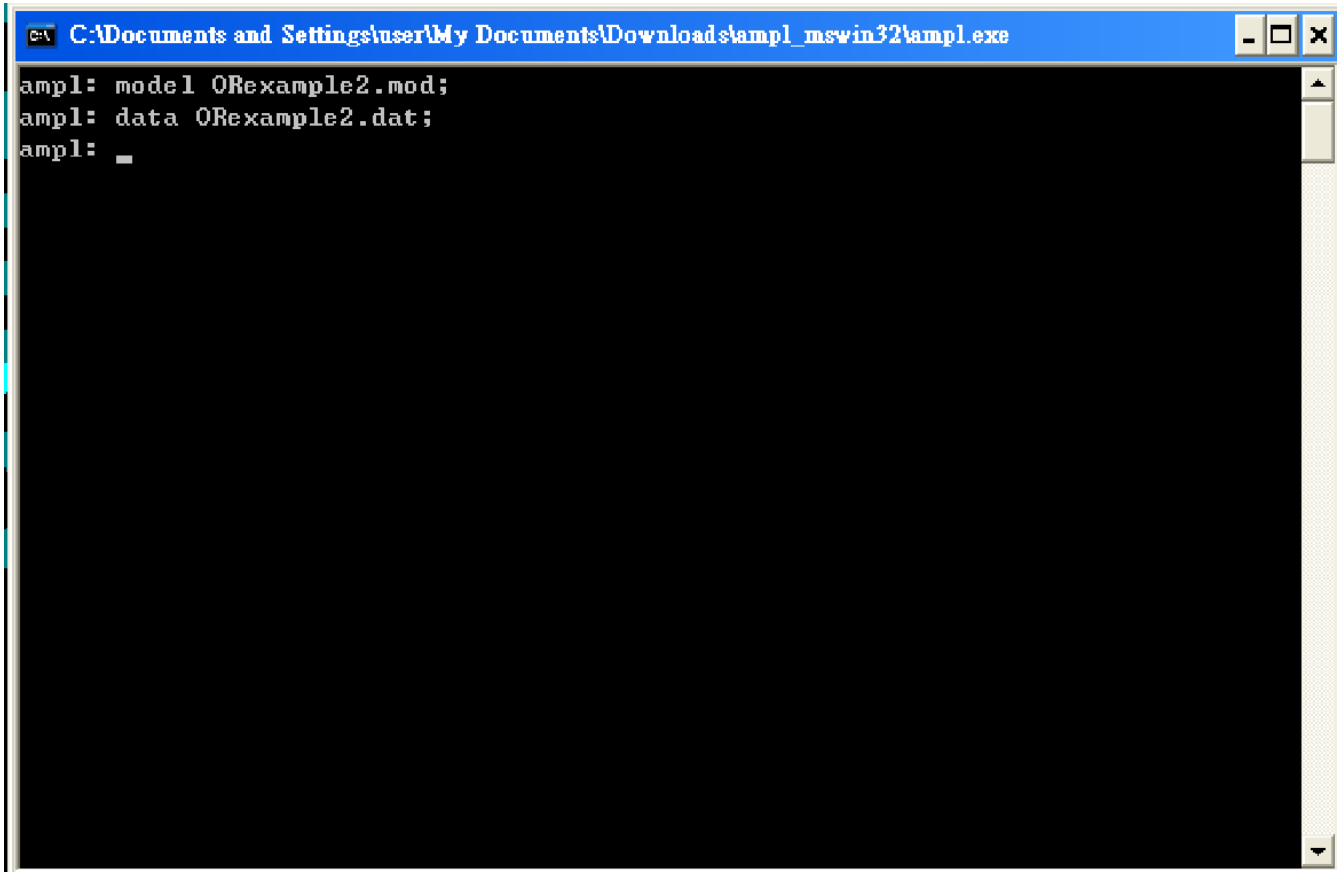
ca1.dat | ca1.mod | ORexample2.mod | ORexample2.dat

```
1  set I;
2  set J;
3  param Ingredient{i in I, j in J};
4  param Cost{i in I};
5  param Lower{j in J};
6  var x{i in I}; # pound of chemical
7
8  minimize Z: sum{i in I} Cost[i] * x[i];
9  subject to Total: sum{i in I} x[i] =1000; #x1+x2+x3+x4 = 1000;
10 subject to Quality{j in J}: sum{i in I} Ingredient[i,j]*x[i] >= Lower[j];
11 subject to Amount1: x[1] >= 0;
12 subject to Amount2: x[2] >= 100;
13 subject to Amount3 : x[3] >= 0;
14 subject to Amount4 : x[4] >= 0;
15
16
```

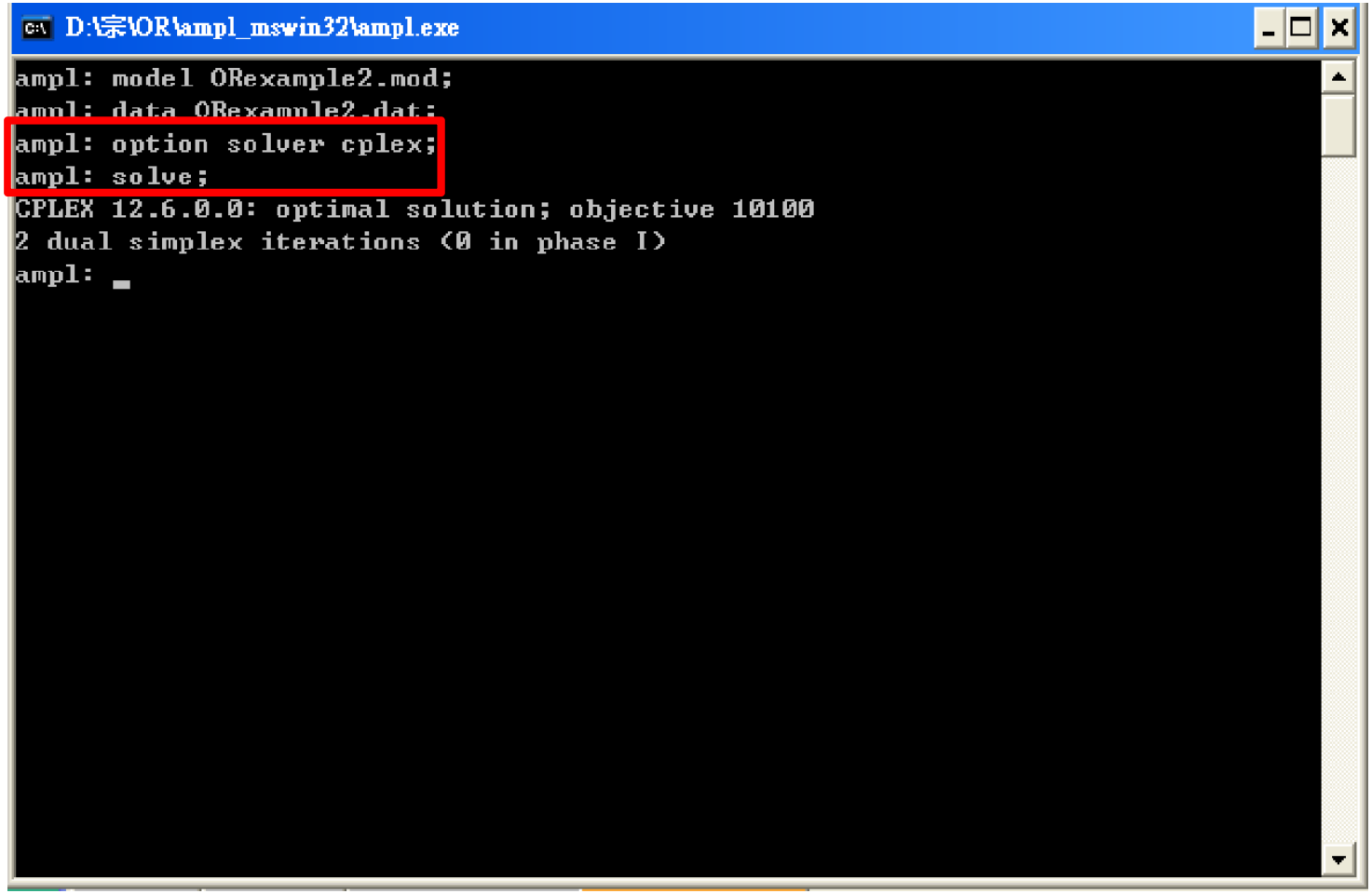
ca1.dat | ca1.mod | ORexample2.mod | ORexample2.dat

```
set I :=  1 2 3 4; # number of chemical
set J := A B C; #number of ingredient

param Ingredient:
        A       B       C   :=
    1   0.04    0.02    0.01
    2   0.06    0.05    0.01
    3   0.1     0.03    0.03
    4   0.11    0.09    0.4;

param Cost :=
    1   8
    2   12
    3   13
    4   15;

param Lower :=
     A  50
     B  40
     C  20;
```

Input file


C:\Documents and Settings\user\My Documents\Downloads\ampl_mswin32\ampl.exe
ampl: model ORexample2.mod;
ampl: data ORexample2.dat;
ampl:

Result


D:\宗\OR\ampl_mswin32\ampl.exe
ampl: model ORexample2.mod;
ampl: data ORexample2.dat;
ampl: option solver cplex;
ampl: solve;
CPLEX 12.6.0.0: optimal solution; objective 10100
2 dual simplex iterations (0 in phase I)
ampl:

Thank you ☺