Common Mathematical Notations and Operations

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Note. Throughout this handout, we use x, y, and z to denote real numbers or vectors, n and m to denote integers, and i, j, and k to denote indices. MS Excel operations/functions are typed `in this way`. If you find anything helpful that should be added into the list, please let me know. Thank you.

1 Mathematical notations

- $\mathbb{N}$ is the set of all natural numbers (positive integers); $\mathbb{Z}$ is the set of all integers; $\mathbb{Q}$ is the set of all rational numbers (which can be written as the ratio of two integers); $\mathbb{R}$ is the set of all real numbers.

- ( ) is a pair of parentheses, [ ] is a pair of square brackets, and { } is a pair of curly brackets.

- $[x, y]$ is the (closed) interval containing all real numbers between x and y, including x and y. We write $z \in [x, y]$ if $x \leq z \leq y$.

- (x, y) is the open interval containing all real numbers between x and y, excluding x and y. We write $z \in (x, y)$ if $x < z < y$.

- $[x, y)$ is the right open interval containing all real numbers between x and y, including x but excluding y. We write $z \in [x, y)$ if $x \leq z < y$.

- $(x, y]$ is the left open interval containing all real numbers between x and y, including y but excluding x. We write $z \in (x, y]$ if $x < z \leq y$.

- A scalar is a single number; a vector is a sequence of numbers. Sometimes we write $x = (x_1, x_2, ..., x_n)$ to represent a vector of length n, where x_i is the ith element/number in vector x.

2 Mathematics operations

- $x + y$, read as "x plus y," means adding x and y to find their summation. E.g., $5 + 7$ is 12. In MS Excel, do this by typing `5 + 7`.

- $x - y$, read as "x minus y," means subtracting y from x to find their difference. E.g., $5 - 7$ is -2. In MS Excel, do this by typing `5 - 7`.

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- xy or $x \times y$, read as "xy" or "x times y," means multiplying x by y to find their product. E.g., $5 \times 7 = 35$. In MS Excel, do this by typing `5 * 7`.

- $\frac{x}{y}$, read as "x divided by y" or "x over y," means dividing x by y to find the ratio of x to y. E.g., $\frac{6}{2} = 3$. In MS Excel, do this by typing `6 / 2`.

- $\mathrm{mod}(n, m)$ is the operation for dividing n by m to find the remainder of this division. This is read as "x modulus y" by some people. E.g., $\mathrm{mod}(10, 3) = 1$. In MS Excel, do this by typing `mod(10, 3)`.

- x^n, read as "x to the power of n," means multiplying x for n times to find its nth power. E.g., $3^5 = 243$. In MS Excel, do this by typing `3 ^ 5` or `power(3, 5)`. In particular, x^2 is read as "x square" while x^3 is read as "x cube."

- $\sqrt{x}$, read as "square root of x," means finding a number y such that $y^2 = x$. E.g., $\sqrt{9} = 3$. In MS Excel, do this by typing `sqrt(9)`.

- x_i, read just as "x i" or "x sub i," means finding the ith element of vector x. E.g., if $x = (10, 11, 12)$, $x_2 = 11$.

- $\sum_{i=1}^{n} x_i$, read as "sum from x_1 to x_n," means to calculate $x_1 + x_2 + \cdots + x_n$. E.g., if $x = (10, 11, 12)$, $\sum_{i=1}^{n} x_i = 33$. In MS Excel, do this by typing `sum(A1:A3)` if the three numbers are contained in cells A1 to A3.

- More generally, $\sum_{i=j}^{k} x_i$, read as "sum from x_j to x_k," means to calculate $x_j + x_{j+1} + \cdots + x_k$ for some numbers $j \geq 1$ and $k \leq n$. E.g., if $x = (10, 11, 12)$, $\sum_{i=2}^{3} x_i = 23$. In MS Excel, do this by typing `sum(A2:A3)` if the two numbers are contained in cells A2 and A3.[1]

- $\lfloor x \rfloor$, read as "floor of x,", means rounding down x to the closest integer no greater than x. E.g., $\lfloor 1.9 \rfloor = 1$. In MS Excel, do this by typing `floor(1.9, 1)`, where the second argument is the multiple to which you want to round.

- $\lceil x \rceil$, read as "ceiling of x," means rounding up x to the closest integer no less than x. E.g., $\lceil 1.1 \rceil = 2$. In R, do this by typing `ceiling(1.1, 1)`, where the second argument is the multiple to which you want to round.

- $|x|$, read as "the absolute value of x," means finding the distance between x and 0. E.g., $|-5| = 5$. In MS Excel, do this by typing `abs(-5)`.

- $n!$, read as "the factorial of n," means finding the product of all positive integers no greater than n. E.g., $3! = 3 \times 2 \times 1 = 6$. In MS Excel, do this by typing `fact(3)`.

- $\max\{x, y\}$ and $\min\{x, y\}$, read as "the maximum of x and y" and "the minimum of x and y," mean finding the larger and smaller one between x and y, respectively. E.g., $\max\{1, 4\} = 4$ and $\min\{-3, 4\} = -3$. In MS Excel, do these by typing `max(x, y)` and `min(x, y)`.

- When x is a vector, $\max\limits_{i=1,\ldots,n}\{x_i\}$ and $\min\limits_{i=1,\ldots,n}\{x_i\}$ are the largest and smallest elements in x, respectively. In MS Excel, put the cells containing the vector into the functions `max()` and `min()`.

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[1] When we have enough spaces, we write $\displaystyle\sum_{i=j}^{k} x_i$.