

Common Mathematical Notations and Operations

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Note. Throughout this handout, we use x , y , and z to denote real numbers or vectors, n and m to denote integers, and i , j , and k to denote indices. MS Excel operations/functions are typed in **this way**. If you find anything helpful that should be added into the list, please let me know. Thank you.

1 Mathematical notations

- \mathbb{N} is the set of all natural numbers (positive integers); \mathbb{Z} is the set of all integers; \mathbb{Q} is the set of all rational numbers (which can be written as the ratio of two integers); \mathbb{R} is the set of all real numbers.
- $()$ is a pair of parentheses, $[]$ is a pair of square brackets, and $\{ \}$ is a pair of curly brackets.
- $[x, y]$ is the (closed) interval containing all real numbers between x and y , including x and y . We write $z \in [x, y]$ if $x \leq z \leq y$.
- (x, y) is the open interval containing all real numbers between x and y , excluding x and y . We write $z \in (x, y)$ if $x < z < y$.
- $[x, y)$ is the right open interval containing all real numbers between x and y , including x but excluding y . We write $z \in [x, y)$ if $x \leq z < y$.
- $(x, y]$ is the left open interval containing all real numbers between x and y , including y but excluding x . We write $z \in (x, y]$ if $x < z \leq y$.
- A scalar is a single number; a vector is a sequence of numbers. Sometimes we write $x = (x_1, x_2, \dots, x_n)$ to represent a vector of length n , where x_i is the i th element/number in vector x .

2 Mathematics operations

- $x + y$, read as “ x plus y ,” means adding x and y to find their summation. E.g., $5 + 7$ is 12. In MS Excel, do this by typing `5 + 7`.
- $x - y$, read as “ x minus y ,” means subtracting y from x to find their difference. E.g., $5 - 7$ is -2 . In MS Excel, do this by typing `5 - 7`.
- xy or $x \times y$, read as “ xy ” or “ x times y ,” means multiplying x by y to find their product. E.g., $5 \times 7 = 35$. In MS Excel, do this by typing `5 * 7`.

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- $\frac{x}{y}$, read as “ x divided by y ” or “ x over y ,” means dividing x by y to find the ratio of x to y . E.g., $\frac{6}{2} = 3$. In MS Excel, do this by typing `6 / 2`.
- $\text{mod}(n, m)$ is the operation for dividing n by m to find the remainder of this division. This is read as “ x modulus y ” by some people. E.g., $\text{mod}(10, 3) = 1$. In MS Excel, do this by typing `mod(10, 3)`.
- x^n , read as “ x to the power of n ,” means multiplying x for n times to find its n th power. E.g., $3^5 = 243$. In MS Excel, do this by typing `3 ^ 5` or `power(3, 5)`. In particular, x^2 is read as “ x square” while x^3 is read as “ x cube.”
- \sqrt{x} , read as “square root of x ,” means finding a number y such that $y^2 = x$. E.g., $\sqrt{9} = 3$. In MS Excel, do this by typing `sqrt(9)`.
- x_i , read just as “ x i ” or “ x sub i ,” means finding the i th element of vector x . E.g., if $x = (10, 11, 12)$, $x_2 = 11$.
- $\sum_{i=1}^n x_i$, read as “sum from x_1 to x_n ,” means to calculate $x_1 + x_2 + \dots + x_n$. E.g., if $x = (10, 11, 12)$, $\sum_{i=1}^3 x_i = 33$. In MS Excel, do this by typing `sum(A1:A3)` if the three numbers are contained in cells A1 to A3.
- More generally, $\sum_{i=j}^k x_i$, read as “sum from x_j to x_k ,” means to calculate $x_j + x_{j+1} + \dots + x_k$ for some numbers $j \geq 1$ and $k \leq n$. E.g., if $x = (10, 11, 12)$, $\sum_{i=2}^3 x_i = 23$. In MS Excel, do this by typing `sum(A2:A3)` if the two numbers are contained in cells A2 and A3.¹
- $\lfloor x \rfloor$, read as “floor of x ,” means rounding down x to the closest integer no greater than x . E.g., $\lfloor 1.9 \rfloor = 1$. In MS Excel, do this by typing `floor(1.9, 1)`, where the second argument is the multiple to which you want to round.
- $\lceil x \rceil$, read as “ceiling of x ,” means rounding up x to the closest integer no less than x . E.g., $\lceil 1.1 \rceil = 2$. In R, do this by typing `ceiling(1.1, 1)`, where the second argument is the multiple to which you want to round.
- $|x|$, read as “the absolute value of x ,” means finding the distance between x and 0. E.g., $|-5| = 5$. In MS Excel, do this by typing `abs(-5)`.
- $n!$, read as “the factorial of n ,” means finding the product of all positive integers no greater than n . E.g., $3! = 3 \times 2 \times 1 = 6$. In MS Excel, do this by typing `fact(3)`.
- $\max\{x, y\}$ and $\min\{x, y\}$, read as “the maximum of x and y ” and “the minimum of x and y ,” mean finding the larger and smaller one between x and y , respectively. E.g., $\max\{1, 4\} = 4$ and $\min\{-3, 4\} = -3$. In MS Excel, do these by typing `max(x, y)` and `min(x, y)`.
- When x is a vector, $\max_{i=1, \dots, n} \{x_i\}$ and $\min_{i=1, \dots, n} \{x_i\}$ are the largest and smallest elements in x , respectively. In MS Excel, put the cells containing the vector into the functions `max()` and `min()`.

¹When we have enough spaces, we write $\sum_{i=j}^k x_i$.