Statistics I, Fall 2012
Suggested Solution for Homework 12

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1. Let μ be the amount of ingredient (in mg) in the product.

   (a) The research hypothesis can be either "the amount of ingredient in the product is larger than 5 mg" or "the amount of ingredient in the product is smaller than 5 mg."

   (b) The statistical hypothesis is

   $$H_0 : \mu = 5$$
   $$H_0 : \mu > 5.$$

   (c) The statistical hypothesis is

   $$H_0 : \mu = 5$$
   $$H_0 : \mu < 5.$$

2. (a) The statistical hypothesis is

   $$H_0 : \mu_1 = \mu_2$$
   $$H_0 : \mu_1 > \mu_2.$$

   (b) Once there is a strong evidence showing that teenagers spend more on fast food than adults, we should make suggestions. Because we should make suggestions only if there is such a strong evidence, $\mu_1 > \mu_2$ should be put in the alternative hypothesis.

   (c) Because the p-value is larger than α in this one-tailed test, we do not reject H_0. There is no strong evidence showing that teenagers spend more on fast food than adults. We should not suggest the government to incentivize more non-fast-food restaurant specifically targeting at teenagers.

3. Let μ be the average willingness-to-pay (in \$) of all consumers.

   (a) The statistical hypothesis is

   $$H_0 : \mu = 60$$
   $$H_0 : \mu > 60.$$

   (b) Let $\overline{X}$ be the sample mean. We need to control the error probability to be α, where the error means rejecting H_0 when H_0 is true. When H_0 is true, the mean of $\overline{X}$ is 60 and rejecting H_0 requires $\overline{X}$ to be much larger than 60. Let d be the required distance (whose value cannot be calculated in this problem), the equation is

   $$\Pr(\overline{X} > 60 + d) = \alpha.$$

   (c) Because the p-value is smaller than α in this one-tailed test, we reject H_0. There is a strong evidence showing that the average willingness-to-pay is higher than \$60. We should increase the unit price.

4. Let μ be the population mean. The population standard deviation is $\sqrt{1600} = 40$.

(a) The statistical hypothesis is

$$H_0\colon \mu = 70$$
$$H_0\colon \mu \neq 70.$$

In this two-tailed test, because $90 > 70$, the p-value is

$$\Pr(\overline{X} > 90) = \Pr\left(Z > \frac{90-70}{40/\sqrt{16}}\right) = \Pr(Z > 2) = 0.0228.$$

(b) The statistical hypothesis is

$$H_0\colon \mu = 70$$
$$H_0\colon \mu > 70.$$

In this one-tailed test, the p-value is

$$\Pr(\overline{X} > 60) = \Pr\left(Z > \frac{60-70}{40/\sqrt{16}}\right) = \Pr(Z > -1) = 0.8413.$$

5. (a) False. The significance level is specified by the researcher.
   (b) False. The p-value does not depend on the specified significance level.
   (c) True.
   (d) False. The critical values does not depend on a p-value.
   (e) True.
   (f) False. While we do not have enough evidence showing that H_a is true, this does not mean that H_0 is true.
   (g) True.
   (h) False. The significance level is the probability of rejecting H_0 when H_0 is true.
   (i) False. It should be the significance level, not the p-value.
   (j) False. This just means that it is more probable to reject H_0.

6. (a) Because the sample mean lies in the rejection region, we reject H_0. With a 5% significance level, there is a strong evidence showing that $\mu < 100$.
   (b) According to Alice's rejection region $(-\infty, 95)$, Alice will reject H_0 if the sample mean is smaller than 100 by at least 5. Therefore, Bob should reject H_0 when the sample mean is larger than 100 by at least 5, Bob's rejection region should be $(105, \infty)$.
   (c) Because the sample mean does not lie in the rejection region, we do not reject H_0. With a 5% significance level, there is no strong evidence showing that $\mu > 100$.
   (d) No. While Alice can conclude (with a 5% significance level) that $\mu < 100$, Bob in fact can conclude nothing.

7. First of all, note that $\Pr(\overline{X} < \mu - x|\mu = \mu_0)$ increases as x decreases. In calculating d', we minimize x subject to the constraint that $\Pr(\overline{X} < \mu - x|\mu = \mu_0) \leq \alpha$. Therefore, the minimum x (denoted as d') is the one that sets $\Pr(\overline{X} < \mu - x|\mu = \mu_0) = \alpha$, otherwise we should further increase x. It then follows $d' = d$.