1. (Three-player Cournot competition) Consider a three-player Cournot competition, in which three firms simultaneously set their supply quantities q_1, q_2, and q_3, the unit price is $a - Q = a - (q_1 + q_2 + q_3)$ for some $a > 0$, and the unit production cost is $c < a$ for all the three firms.

   (a) Find the equilibrium supply quantities. Find all if there are more than one.

   (b) When the number of firms goes from two to three, does the equilibrium supply of a single firm increase or decrease? Intuitively explain why.

   (c) What is the equilibrium profit earned by a firm when the number of firms approaches infinity?

2. (Bertrand competition for heterogeneous products) Consider the following game which is called the *Bertrand competition* (for heterogeneous products). Two firms, 1 and 2, simultaneously set prices p_1 and p_2 for two substitutes. Given these prices, firm 1 sells $q_1 = a - p_1 + bp_2$ and firm 2 sells $q_2 = a - p_2 + bp_1$, where $a > 0$ and $b \in [0, 1]$. There is a unit production cost $c < a$ for both firms. Suppose each firm wants to maximize its own profit.

   (a) Intuitively explain why $b \in [0, 1]$. Is a negative or greater-than-one b reasonable?

   (b) Find the equilibrium prices. Find all if there are more than one.

   (c) How do a, b, and c affect the equilibrium prices? Intuitively explain why.

(d) Suppose the two firms cooperate and determine a single retail price p to maximize the sum of their profits. In this case, the demand of firm i is $q_i = a - p + bp = a - (1 - b)p$ for $i = 1, 2$. Find the optimal price in this case. Is it higher or lower than the equilibrium prices under decentralization? Intuitively explain why.

3. (Hotelling line) Consider the following game in which two firms compete in a so-called *Hotelling line.* Two firms, 1 and 2, simultaneously set there store locations x_1 and x_2 within a line segment $[0, 1]$. Consumers spread on the line segment uniformly. Once the locations are set, a consumer will go to the store that is closer to her; if the two stores are equally close, she will go to either one with probability $\frac{1}{2}$. Each firm wants to maximize the expected number of consumers going into its store.

   (a) Find the equilibrium locations. Find all if there are more than one.

   (b) If consumers do not spread uniformly, what should the two firms do?

4. (Ultimatum game) Consider the following dynamic game which is called the *ultimatum game*. In an ultimatum game, player 1 first decides how to share \$1 with player 2 by making an offer $x \in [0, 1]$. Player 2 then chooses to either accept or reject the offer. If he accepts, he earns x and player 1 earns $(1 - s)$. If he rejects, both of them earns 0. Each of them wants to maximize her or his share of that \$1.

   (a) Draw the game tree by using a shaded area to depict a set of infinitely many actions.

   (b) Find the equilibrium outcome. Is there any assumption you need to make?

(c) Suppose we must maintain that player 1 offers first and then player 2 makes a decision, how may we modify this game to achieve a fair allocation (i.e., each of them get \$0.5 in equilibrium?

5. (Horizontal and vertical integration) In the Cournot game, we find that a *horizontal integration* benefits the firms but harms consumers. How about a *vertical integration* that integrating the two firms in our supply chain pricing problem? Mathematically show it and intuitively explain why.