Information Economics

The Two-type Screening Model

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Road map

- **Introduction to screening**.
- First best with complete information.
- Incentives and the revelation principle.
- Finding the second best.
- Appendix: Proof of the revelation principle.

Monopoly pricing

- Let's start with a **monopoly pricing** problem.
- Imagine that you produce and sell one product.
- You are the only one who are able to produce and sell this product.
- How would you price your product to maximize your profit?

Monopoly pricing

- Suppose that consumers' **valuation** θ (willingness-to-pay) lie uniformly within $[0,1]$.
- A consumer's utility is $\theta - p$, where θ is his valuation and p is the price.
- Given a price p, those with $\theta \in [p,1]$ will buy the product. The demand function is $q(p) = 1 - p$. The seller will solve

$$\pi^* = \max\ (1-p)p \quad \Rightarrow \quad p^* = \frac{1}{2} \quad \Rightarrow \quad \pi^* = \frac{1}{4}.$$

- Let's calculate consumer surplus:
  - A consumer whose $\theta > p$ will earn $\theta - p$ as the surplus.
  - The total surplus earned by all consumers is

$$\int_p^1 (\theta - p) d\theta = \frac{1}{2}(1-p)^2.$$

  - As $p^* = \frac{1}{2}$, the consumer surplus is $\frac{1}{8}$.

Monopoly pricing

- Here comes a critic:
  - "Some people are willing to pay more, but your price is too low!"
  - "Some potential sales are lost because your price is too high!"
- His (useless) suggestion is:
  - "Who told you that you may set only one price?"
  - "Ask them how they like the product and charge differently!"
- Does that work?
- **Price discrimination** is impossible if consumers' valuations are completely hidden to you.
- If you can see the valuation, you will charge each consumer his valuation. This is **perfect price discrimination**.

Information asymmetry and inefficiency

- Let's calculate the monopolist's profit and consumer surplus under perfect price discrimination.
  - Monopolist's profit: $\int_0^1 \theta d\theta = \frac{1}{2}$.
  - Consumer surplus: 0.
  - Social welfare: $\frac{1}{2} + 0 = \frac{1}{2}$.
- **Information asymmetry** causes **inefficiency**.
  - Social welfare decreases under information asymmetry.
  - However, it **protects** the agent.
- Note that decentralization does not necessarily cause inefficiency. Here information asymmetry is the reason!

Adverse selection: screening

- Consider the following buyer-seller relationship:
  - A manufacturer decides to buy a critical component of its product.
  - She finds a supplier that supplies this part.
  - Two kinds of technology can produce this component with **different unit costs**.
  - When a manufacturer faces the supplier, she **does not know** which kind of technology is owned by the supplier.
  - How much should the manufacturer pay for the part?
- The difficulty is:
  - If I know the supplier's cost is low, I will be able to ask for a low price.
  - However, if I ask him, he will **always** claim that his cost is high!
- The manufacturer wants to find a way to **screen** the supplier's **type**.

Adverse selection: screening

- An agent always want to **hide his type** to get bargaining power!
  - The "type" of an agent is a part of his **utility function** that is **private**.
- In the previous example:
  - The manufacturer is the principal.
  - The supplier is the agent.
  - The unit production cost is the agent's type.
- More examples:
  - A retailer does not know how to charge an incoming consumer because the consumer's **willingness-to-pay** is hidden.
  - An adviser does not know how to assign reading assignments to her graduate students because the students' **reading ability** is hidden.

Mechanism design

- One way to deal with agents' private information is to become more knowledgeable.
- When such an information-based approach is not possible, one way to screen a type is through **mechanism design**.
  - Or in the business world, **contract design**.
  - The principal will design a mechanism/contract that can "find" the agent's type.
- We will start from the easiest case: The agent's type has only two possible values. In this case, there are **two types** of agents.

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The two-type model

- In general, no consumer would be willing to tell you his preference.
- Consider the easiest case with valuation heterogeneity: There are **two** kinds of consumers.
- When obtaining q units by paying T, a **type-θ** consumer's utility is

$$u(q, T, \theta) = \theta v(q) - T.$$

  - $\theta \in \{\theta_\mathrm{L}, \theta_\mathrm{H}\}$ where $\theta_\mathrm{L} < \theta_\mathrm{H}$. θ is the consumer's **private** information.
  - $v(q)$ is strictly increasing and strictly concave. $v(0) = 0$.
  - A **high-type** (**type-H**) consumer's θ is θ_H.
  - A **low-type** (**type-L**) consumer's θ is θ_L.
  - The seller believes that $\Pr(\theta = \theta_\mathrm{L}) = \beta = 1 - \Pr(\theta = \theta_\mathrm{H})$.
- The unit production cost of the seller is c. $c < \theta_\mathrm{L}$.
- By selling q units and receiving T, the seller earns $T - cq$.
- How would you price your product to maximize your expected profit?

The two-type model with complete information

- Under complete information, the seller sees the consumer's type.
- Facing a type-H consumer, the seller solves

$$\begin{aligned} \max_{q_{\mathrm{H}} \geq 0, T_{\mathrm{H}} \text{ urs.}} \quad & T_{\mathrm{H}} - c q_{\mathrm{H}} \\ \text{s.t.} \quad & \theta_{\mathrm{H}} v(q_{\mathrm{H}}) - T_{\mathrm{H}} \geq 0. \end{aligned}$$

- To solve this problem, note that the constraint must be **binding** (i.e., being an equality) at any optimal solution.
  - Otherwise we will increase T_{H}.
  - Any optimal solution satisfies $\theta_{\mathrm{H}} v(q_{\mathrm{H}}) - T_{\mathrm{H}} = 0$.
  - The problem is equivalent to

$$\max_{q_{\mathrm{H}} \geq 0} \theta_{\mathrm{H}} v(q_{\mathrm{H}}) - c q_{\mathrm{H}}.$$

  - The FOC characterize the optimal quantity $\tilde{q}_{\mathrm{H}}$: $\theta_{\mathrm{H}} v'(\tilde{q}_{\mathrm{H}}) = c$.
  - The optimal transfer is $\tilde{T}_{\mathrm{H}} = \theta_{\mathrm{H}} v(\tilde{q}_{\mathrm{H}})$.

The two-type model with complete information

- For the type-i consumer, the **first-best** solution $(\tilde{q}_i, \tilde{T}_i)$ satisfies

$$\theta_i v'(\tilde{q}_i) = c \quad \text{and} \quad \tilde{T}_i = \theta_i v(\tilde{q}_i) \quad \forall i \in \{\text{L}, \text{H}\}$$

- The **rent** of the consumer is his surplus of trading.
- For either type, the consumer receives **no rent**!
- The seller extracts all the rents from the consumer.
- Next we will introduce the optimal pricing plan under information asymmetry and, of course, deliver some insights to you.

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Pricing under information asymmetry

- When the valuation is hidden, the first-best plan does not work.
  - You cannot make an offer (a pair of q and T) according to his type.
- How about offering a **menu** of two contracts, $\{(\tilde{q}_{\mathrm{L}}, \tilde{T}_{\mathrm{L}}), (\tilde{q}_{\mathrm{H}}, \tilde{T}_{\mathrm{H}})\}$, for the consumer to select?
- You **cannot expect** the type-i consumer to select $(\tilde{q}_i, \tilde{T}_i)$, $i \in \{\mathrm{L}, \mathrm{H}\}$!
  - Both types will select $(\tilde{q}_{\mathrm{L}}, \tilde{T}_{\mathrm{L}})$.
  - In particular, the type-H consumer will earn a **positive rent**:

$$
\begin{aligned}
u(\tilde{q}_{\mathrm{L}}, \tilde{T}_{\mathrm{L}}, \theta_{\mathrm{H}}) &= \theta_{\mathrm{H}} v(\tilde{q}_{\mathrm{L}}) - \tilde{T}_{\mathrm{L}} \\
&= \theta_{\mathrm{H}} v(\tilde{q}_{\mathrm{L}}) - \theta_{\mathrm{L}} v(\tilde{q}_{\mathrm{L}}) \\
&= (\theta_{\mathrm{H}} - \theta_{\mathrm{L}}) v(\tilde{q}_{\mathrm{L}}) > 0.
\end{aligned}
$$

- It turns out that the first-best solution is not optimal under information asymmetry.

Incentive compatibility

- The first-best menu $\{(\tilde{q}_{\mathrm{L}}, \tilde{T}_{\mathrm{L}}), (\tilde{q}_{\mathrm{H}}, \tilde{T}_{\mathrm{H}})\}$ is said to be **incentive-incompatible**:
  - The type-H consumer has an incentive to **hide** his type and **pretend** to be a type-L one.
  - This fits our common intuition!
- A menu is **incentive-compatible** if different types of consumers will select different contracts.
  - An incentive-compatible contract induces **truth-telling**.
  - According to his selection, we can identify his type!
- How to make a menu incentive-compatible?

Incentive-compatible menu

- Suppose a menu $\{(q_{\mathrm{L}}, T_{\mathrm{L}}), (q_{\mathrm{H}}, T_{\mathrm{H}})\}$ is incentive-compatible.
  - The type-H consumer will select $(q_{\mathrm{H}}, T_{\mathrm{H}})$, i.e.,

$$\theta_{\mathrm{H}} v(q_{\mathrm{H}}) - T_{\mathrm{H}} \geq \theta_{\mathrm{H}} v(q_{\mathrm{L}}) - T_{\mathrm{L}}.$$

  - The type-L consumer will select $(q_{\mathrm{L}}, T_{\mathrm{L}})$, i.e.,

$$\theta_{\mathrm{L}} v(q_{\mathrm{L}}) - T_{\mathrm{L}} \geq \theta_{\mathrm{L}} v(q_{\mathrm{H}}) - T_{\mathrm{H}}.$$

- The above two constraints are called the **incentive-compatibility constraints** (IC constraints) or **truth-telling** constraints.
- If the seller wants to do business with both types, she also needs the **individual-rationality constraints** (IR constraints) or **participation** constraints:

$$\theta_i v(q_i) - T_i \geq 0 \quad \forall i \in \{\mathrm{L}, \mathrm{H}\}.$$

- The seller may offer an incentive-compatible menu. But is it optimal?

Inducing truth-telling is optimal

- Among all possible pricing schemes (or mechanisms, in general), some are incentive compatible while some are not.
  - The first-best menu is not; an incentive compatible menu is.
  - Should we induce truth-telling?
- **revelation principle**: Yes!
  - Contributors of the revelation principle include three Nobel Laureates: James Mirrlees (1996), Eric Maskin (2007), and Roger Myerson (2007).
  - There are other contributors.
  - Related works were published in 1970s.
- The revelation principle tells us "At least one incentive-compatible mechanism is optimal."[1]
  - We may restrict our attentions to incentive-compatible menus!
  - The problem then becomes tractable.

[1] A nonrigorous proof is provided in the appendix.

Reducing the search space

- We only need to search among menus that can induce truth-telling.
  - Different types of consumers should select different contracts.
  - As we have only two consumers, two contracts are sufficient.
  - One is not enough and three is too many!
- The problem to solve is

$$\begin{aligned}
\max_{q_{\mathrm{H}}, T_{\mathrm{H}}, q_{\mathrm{L}}, T_{\mathrm{L}}} \quad & \beta\Big[T_{\mathrm{L}} - cq_{\mathrm{L}}\Big] + (1-\beta)\Big[T_{\mathrm{H}} - cq_{\mathrm{H}}\Big] && \text{(OBJ)} \\
\text{s.t.} \quad & \theta_{\mathrm{H}} v(q_{\mathrm{H}}) - T_{\mathrm{H}} \geq \theta_{\mathrm{H}} v(q_{\mathrm{L}}) - T_{\mathrm{L}} && \text{(IC-H)} \\
& \theta_{\mathrm{L}} v(q_{\mathrm{L}}) - T_{\mathrm{L}} \geq \theta_{\mathrm{L}} v(q_{\mathrm{H}}) - T_{\mathrm{H}} && \text{(IC-L)} \\
& \theta_{\mathrm{H}} v(q_{\mathrm{H}}) - T_{\mathrm{H}} \geq 0 && \text{(IR-H)} \\
& \theta_{\mathrm{L}} v(q_{\mathrm{L}}) - T_{\mathrm{L}} \geq 0. && \text{(IR-L)}
\end{aligned}$$

  - IC constraints ensure truth-telling. IR constraints ensure participation.

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Solving the two-type problem

- Below we will introduce the standard way of solving the standard two-type problem[2]

$$
\begin{aligned}
\max_{q_{\mathrm{H}}, T_{\mathrm{H}}, q_{\mathrm{L}}, T_{\mathrm{L}}} \quad & \beta\Big[T_{\mathrm{L}} - cq_{\mathrm{L}}\Big] + (1-\beta)\Big[T_{\mathrm{H}} - cq_{\mathrm{H}}\Big] && \text{(OBJ)} \\
\text{s.t.} \quad & \theta_{\mathrm{H}} v(q_{\mathrm{H}}) - T_{\mathrm{H}} \geq \theta_{\mathrm{H}} v(q_{\mathrm{L}}) - T_{\mathrm{L}} && \text{(IC-H)} \\
& \theta_{\mathrm{L}} v(q_{\mathrm{L}}) - T_{\mathrm{L}} \geq \theta_{\mathrm{L}} v(q_{\mathrm{H}}) - T_{\mathrm{H}} && \text{(IC-L)} \\
& \theta_{\mathrm{H}} v(q_{\mathrm{H}}) - T_{\mathrm{H}} \geq 0 && \text{(IR-H)} \\
& \theta_{\mathrm{L}} v(q_{\mathrm{L}}) - T_{\mathrm{L}} \geq 0. && \text{(IR-L)}
\end{aligned}
$$

- The key is that we want to **analytically** solve the problem.
  - With the analytical solution, we may generate some insights.

[2] Technically, we should also have nonnegativity constraints $q_{\mathrm{H}} \geq 0$ and $q_{\mathrm{L}} \geq 0$. To make the presentation concise, I will hide them.

Step 1: Monotonicity

- By adding the two IC constraints

$$\theta_{\mathrm{H}} v(q_{\mathrm{H}}) - T_{\mathrm{H}} \geq \theta_{\mathrm{H}} v(q_{\mathrm{L}}) - T_{\mathrm{L}}$$

and

$$\theta_{\mathrm{L}} v(q_{\mathrm{L}}) - T_{\mathrm{L}} \geq \theta_{\mathrm{L}} v(q_{\mathrm{H}}) - T_{\mathrm{H}},$$

we obtain

$$\begin{aligned}
& \theta_{\mathrm{H}} v(q_{\mathrm{H}}) + \theta_{\mathrm{L}} v(q_{\mathrm{L}}) \geq \theta_{\mathrm{H}} v(q_{\mathrm{L}}) + \theta_{\mathrm{L}} v(q_{\mathrm{H}}) \\
\Rightarrow\ & (\theta_{\mathrm{H}} - \theta_{\mathrm{L}}) v(q_{\mathrm{H}}) \geq (\theta_{\mathrm{H}} - \theta_{\mathrm{L}}) v(q_{\mathrm{L}}) \\
\Rightarrow\ & v(q_{\mathrm{H}}) \geq v(q_{\mathrm{L}}) \\
\Rightarrow\ & q_{\mathrm{H}} \geq q_{\mathrm{L}}.
\end{aligned}$$

- This is the **monotonicity** condition: In an incentive-compatible menu, the high-type consumer consume more.
  - Intuition: The high-type consumer prefers a high consumption.

Step 2: (IR-H) is redundant

- (IC-H) and (IR-L) imply that (IR-H) is redundant:

$$\begin{aligned} \theta_{\rm H} v(q_{\rm H}) - T_{\rm H} &\geq \theta_{\rm H} v(q_{\rm L}) - T_{\rm L} && \text{(IC-H)} \\ &> \theta_{\rm L} v(q_{\rm L}) - T_{\rm L} && (\theta_{\rm H} > \theta_{\rm L}) \\ &\geq 0. && \text{(IR-L)} \end{aligned}$$

- The high-type consumer earns **a positive rent**. Full surplus extraction is impossible under information asymmetry.
- The problem reduces to

$$\begin{aligned} \max_{q_{\rm H}, T_{\rm H}, q_{\rm L}, T_{\rm L}} \quad & \beta\Big[T_{\rm L} - c q_{\rm L}\Big] + (1-\beta)\Big[T_{\rm H} - c q_{\rm H}\Big] && \text{(OBJ)} \\ \text{s.t.} \quad & \theta_{\rm H} v(q_{\rm H}) - T_{\rm H} \geq \theta_{\rm H} v(q_{\rm L}) - T_{\rm L} && \text{(IC-H)} \\ & \theta_{\rm L} v(q_{\rm L}) - T_{\rm L} \geq \theta_{\rm L} v(q_{\rm H}) - T_{\rm H} && \text{(IC-L)} \\ & \theta_{\rm L} v(q_{\rm L}) - T_{\rm L} \geq 0. && \text{(IR-L)} \end{aligned}$$

Step 3: Ignore (IC-L)

- Let's "guess" that (IC-L) will be redundant and ignore it for a while.
  - Intuition: The low-type consumer **has no incentive** to pretend that he really likes the product.
  - We will verify that the optimal solution of the relaxed program indeed satisfies (IC-L).
- The problem reduces to

$$\begin{aligned} \max_{q_{\mathrm{H}}, T_{\mathrm{H}}, q_{\mathrm{L}}, T_{\mathrm{L}}} \quad & \beta \Big[T_{\mathrm{L}} - c q_{\mathrm{L}} \Big] + (1-\beta) \Big[T_{\mathrm{H}} - c q_{\mathrm{H}} \Big] && \text{(OBJ)} \\ \text{s.t.} \quad & \theta_{\mathrm{H}} v(q_{\mathrm{H}}) - T_{\mathrm{H}} \geq \theta_{\mathrm{H}} v(q_{\mathrm{L}}) - T_{\mathrm{L}} && \text{(IC-H)} \\ & \theta_{\mathrm{L}} v(q_{\mathrm{L}}) - T_{\mathrm{L}} \geq 0. && \text{(IR-L)} \end{aligned}$$

Step 4: Remaining constraints bind at optimality

$$\begin{aligned}
\max_{q_{\rm H}, T_{\rm H}, q_{\rm L}, T_{\rm L}} \quad & \beta \Big[T_{\rm L} - c q_{\rm L} \Big] + (1-\beta) \Big[T_{\rm H} - c q_{\rm H} \Big] && \text{(OBJ)} \\
\text{s.t.} \quad & \theta_{\rm H} v(q_{\rm H}) - T_{\rm H} \geq \theta_{\rm H} v(q_{\rm L}) - T_{\rm L} && \text{(IC-H)} \\
& \theta_{\rm L} v(q_{\rm L}) - T_{\rm L} \geq 0. && \text{(IR-L)}
\end{aligned}$$

- (IC-H) must be **binding** at any optimal solution:
  - The seller wants to increase $T_{\rm H}$ as much as possible.
- (IR-L) must also be **binding** at any optimal solution:
  - The seller wants to increase $T_{\rm L}$ as much as possible.
  - Note that increasing $T_{\rm L}$ makes (IC-H) more relaxed rather than tighter.
- Note that if we did not ignore (IC-L), i.e.,

$$\theta_{\rm L} v(q_{\rm L}) - T_{\rm L} \geq \theta_{\rm L} v(q_{\rm H}) - T_{\rm H},$$

we cannot claim that (IR-L) is binding!

Step 5: Removing the transfers

- The problem reduces to

$$\begin{aligned} \max_{q_{\mathrm{H}}, T_{\mathrm{H}}, q_{\mathrm{L}}, T_{\mathrm{L}}} \quad & \beta\Big[T_{\mathrm{L}} - cq_{\mathrm{L}}\Big] + (1-\beta)\Big[T_{\mathrm{H}} - cq_{\mathrm{H}}\Big] && \text{(OBJ)} \\ \text{s.t.} \quad & \theta_{\mathrm{H}} v(q_{\mathrm{H}}) - T_{\mathrm{H}} = \theta_{\mathrm{H}} v(q_{\mathrm{L}}) - T_{\mathrm{L}} && \text{(IC-H)} \\ & \theta_{\mathrm{L}} v(q_{\mathrm{L}}) - T_{\mathrm{L}} = 0. && \text{(IR-L)} \end{aligned}$$

- Therefore, we may remove the two constraints and replace T_{L} and T_{H} in (OBJ) by $\theta_{\mathrm{L}} v(q_{\mathrm{L}})$ and $\theta_{\mathrm{H}} v(q_{\mathrm{H}}) - \theta_{\mathrm{H}} v(q_{\mathrm{L}}) + \theta_{\mathrm{L}} v(q_{\mathrm{L}})$, respectively.
- The problem reduces to an **unconstrained** problem

$$\begin{aligned} \max_{q_{\mathrm{H}}, q_{\mathrm{L}}} \quad & \beta\Big[\theta_{\mathrm{L}} v(q_{\mathrm{L}}) - cq_{\mathrm{L}}\Big] \\ & + (1-\beta)\Big[\theta_{\mathrm{H}} v(q_{\mathrm{H}}) - \theta_{\mathrm{H}} v(q_{\mathrm{L}}) + \theta_{\mathrm{L}} v(q_{\mathrm{L}}) - cq_{\mathrm{H}}\Big]. \end{aligned}$$

Step 6: Solving the unconstrained problem

- To solve

$$\max_{q_{\mathrm{H}}, q_{\mathrm{L}}} \beta\Big[\theta_{\mathrm{L}} v(q_{\mathrm{L}}) - c q_{\mathrm{L}}\Big] + (1-\beta)\Big[\theta_{\mathrm{H}} v(q_{\mathrm{H}}) - c q_{\mathrm{H}} - (\theta_{\mathrm{H}} - \theta_{\mathrm{L}}) v(q_{\mathrm{L}})\Big],$$

  note that because $v(\cdot)$ is strictly concave, the reduced objective function is strictly concave in q_{H} and q_{L}.
- If $\frac{\theta_{\mathrm{H}} - \theta_{\mathrm{L}}}{\theta_{\mathrm{H}}} < \beta$, the **second-best** solution $\{(q_L^*, T_L^*), (q_H^*, T_H^*)\}$ satisfies the FOC:[3]

$$\theta_{\mathrm{H}} v'(q_H^*) = c \quad \text{and} \quad \theta_{\mathrm{L}} v'(q_L^*) = c\left[\frac{1}{1 - \left(\frac{1-\beta}{\beta} \frac{\theta_{\mathrm{H}} - \theta_{\mathrm{L}}}{\theta_{\mathrm{L}}}\right)}\right].$$

[3] If $\frac{\theta_{\mathrm{H}} - \theta_{\mathrm{L}}}{\theta_{\mathrm{H}}} \geq \beta$, $q_L^* = 0$ and q_H^* still satisfies $\theta_{\mathrm{H}} v'(q_H^*) = c$.

Step 7: Verifying that (IC-L) is satisfied

- To verify that (IC-L) is satisfied, we apply

$$T_{\mathrm{L}} = \theta_{\mathrm{L}} v(q_{\mathrm{L}}) \quad \text{and} \quad T_{\mathrm{H}} = \theta_{\mathrm{H}} v(q_{\mathrm{H}}) - (\theta_{\mathrm{H}} - \theta_{\mathrm{L}}) v(q_{\mathrm{L}}).$$

- With this, (IC-L)

$$\theta_{\mathrm{L}} v(q_{\mathrm{L}}) - T_{\mathrm{L}} \geq \theta_{\mathrm{L}} v(q_{\mathrm{H}}) - T_{\mathrm{H}}$$

  is equivalent to

$$0 \geq -(\theta_{\mathrm{H}} - \theta_{\mathrm{L}}) \Big[v(q_{\mathrm{H}}) - v(q_{\mathrm{L}}) \Big].$$

  With the monotonicity condition, (IC-L) is satisfied.

Inefficient consumption levels

- Recall that the first-best consumption levels $\tilde{q}_{\mathrm{L}}$ and $\tilde{q}_{\mathrm{H}}$ satisfy

$$\theta_{\mathrm{H}} v'(\tilde{q}_{\mathrm{H}}) = c \quad \text{and} \quad \theta_{\mathrm{L}} v'(\tilde{q}_{\mathrm{L}}) = c.$$

  Moreover, the second-best consumption levels satisfy

$$\theta_{\mathrm{H}} v'(q_H^*) = c \quad \text{and} \quad \theta_{\mathrm{L}} v'(q_L^*) = c \left[\frac{1}{1 - \left(\frac{1-\beta}{\beta} \frac{\theta_{\mathrm{H}} - \theta_{\mathrm{L}}}{\theta_{\mathrm{L}}} \right)} \right] > c.$$

- The high-type consumer consumes the **first-best** amount.
- For the low-type consumer, $v'(\tilde{q}_{\mathrm{L}}) = \frac{c}{\theta_{\mathrm{L}}} < v'(q_L^*)$. As $v(\cdot)$ is strictly concave (so $v'(\cdot)$ is decreasing), $q_L^* < \tilde{q}_{\mathrm{L}}$.
- The low-type consumer consumes **less** than the first-best amount.
  - Information asymmetry causes inefficiency.
  - The consumption will only decrease. It will not become larger. Why?

Cost of inducing truth-telling

- We have $q_L^* < \tilde{q}_{\mathrm{L}}$. Why do we decrease q_{L}?
  - Recall that under the first-best menu, the high-type consumer pretends to have a low valuation and earns $(\theta_{\mathrm{H}} - \theta_{\mathrm{L}})v(\tilde{q}_{\mathrm{L}}) > 0$.
  - Because he prefers a high consumption level, we must **cut down** q_{L} to make him **unwilling to lie**.
  - Cutting down q_L^* is to cut down his information rent!
- Regarding the consumer surplus:
  - In equilibrium, the low-type consumer earns $\theta_{\mathrm{L}} v(q_L^*) - T_L^* = 0$.
  - However, the high-type consumer earns

$$\theta_{\mathrm{H}} v(q_H^*) - T_H^* = (\theta_{\mathrm{H}} - \theta_{\mathrm{L}})v(q_L^*) > 0.$$

  - The high-type consumer earns a positive **information rent**.
  - The agent earns a positive rent **in expectation**.
- Note that the high-type consumer's rent depends on q_L^*.

Summary

- We discussed a two-type monopoly pricing problem.
- We found the first-best and second-best mechanisms.
  - First-best: with complete information.
  - Second-best: under information asymmetry.
  - Thanks to the revelation principle!
- For the second-best solution:
  - **Monotonicity**: The high-type consumption level is higher.
  - **Efficiency at top**: The high-type consumption level is efficient.
  - **No rent at bottom**: The low-type consumer earns no rent.
- Information asymmetry **protects the agent**.
  - But it hurts the principal and social welfare.

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The idea of the revelation principle

- In general, the principal designs a mechanism for the agent(s).
  - The mechanism specifies a game rule. Agents act according to the rules.
- When agents have private types, there are two kinds of mechanisms.
- Under an **indirect mechanism**:
  - The principal specifies a function mapping agents' actions to payoffs.
  - Each agent, based on his type and his belief on other agents' types, acts to maximize his expected utilities.
- Under a **direct mechanism**:
  - The principal specifies a function mapping agents' **reported types** to actions and payoffs.
  - Each agent, based on his type and his belief on other agents' types, **reports a type** to maximize his expected utilities.
- If a direct mechanism can reveal agents' types (i.e., making all agents report truthfully), it is a **direct revelation mechanism**.

The idea of the revelation principle

Proposition 1 (Revelation principle)

Given any equilibrium of any given indirect mechanism, there is a direct revelation mechanism under which the equilibrium is equivalent to the given one: In the two equilibria, agents do the same actions.

- The idea is to "imitate" the given equilibrium.
- The given equilibrium specifies each agent's (1) strategy to map his type to an action and (2) his expected payoff.
- We may "construct" a direct mechanism as follows:
  - Given any type report (some types may be false), find the **corresponding actions and payoffs** in the given equilibrium as if the agents' types are really as reported.
  - Then assign **exactly those actions and payoffs** to agents.
- If the agents all report truthfully under the direct mechanism, they are receiving exactly what they receive in the given equilibrium. Therefore, under this direct mechanism no one deviates.