1. In our two-type monopoly pricing problem, let $\{(\tilde{q}_H, \tilde{t}_H), (\tilde{q}_L, \tilde{t}_L)\}$ be the first-best menu and $\{(q_H^*, t_H^*), (q_L^*, t_L^*)\}$ be the second-best menu, where q is the consumption and t is the transfer.

   (a) Intuitively explain why $q_L^* < q_H^*$, i.e., why in the second-best menu, the seller will offer the low-type consumer a lower consumption level.

   (b) Intuitively explain why $q_L^* < \tilde{q}_L$, i.e., why the seller wants to cut down the low-type consumption level when there is information asymmetry.

   (c) How about $\tilde{q}_L$ and q_H^*? Which one is bigger?

2. In our two-type monopoly pricing problem, a contract consists of a quantity q and a transfer (fixed payment) t. However, when we are discussing about efficiency, we only focus on q. Why don't we consider t, e.g., compare whether t is higher or lower with information asymmetry or compare the transfers intended for the two types of consumers?

3. Consider the monopoly pricing problem discussed in the lecture videos. The first-best consumption levels $\tilde{q}_{\mathrm{L}}$ and $\tilde{q}_{\mathrm{H}}$ satisfy

$$\theta_{\mathrm{H}} v'(\tilde{q}_{\mathrm{H}}) = c \quad \text{and} \quad \theta_{\mathrm{L}} v'(\tilde{q}_{\mathrm{L}}) = c.$$

Moreover, the second-best consumption levels satisfy

$$\theta_{\mathrm{H}} v'(q_H^*) = c \quad \text{and} \quad \theta_{\mathrm{L}} v'(q_L^*) = c\left[\frac{1}{1-(\frac{1-\beta}{\beta}\frac{\theta_{\mathrm{H}}-\theta_{\mathrm{L}}}{\theta_{\mathrm{L}}})}\right]$$

if $\frac{\theta_{\mathrm{H}}-\theta_{\mathrm{L}}}{\theta_{\mathrm{H}}} < \beta$ or $\theta_{\mathrm{H}} v'(q_{\mathrm{H}}^*) = c$ and $q_{\mathrm{L}}^* = 0$ otherwise.

(a) How do q_L^* and q_H^* change when c changes?

(b) Suppose $q_L^* > 0$, how do q_L^* and q_H^* change when β changes?

(c) When β becomes larger, it is more or less likely for $q_L^* = 0$?

(d) When $\frac{\theta_L}{\theta_H}$ becomes larger, it is more or less likely for $q_L^* = 0$?

4. A retailer is buying a product from a supplier, which may produce the product at a unit cost θ_L with probability β or θ_H with probability $1-\beta$. Assume $\theta_L < \theta_H$ and such a cost is the supplier's private information.

   We refer to a pair of transfer and quantity (t, q) as a contract. For example, if a supplier chooses $(t, q) = (500, 5)$, the retailer will buy 5 units from the supplier and pays \$500 to the supplier. Therefore, if a type-i supplier chooses a contract (t, q), his profit is $t - \theta_i q$. The retailer generates sales revenues by selling those products she obtains. Assume that the sales revenue is a function of the number of products she has and is denoted as $v(q)$, which is strictly increasing and strictly concave. Therefore, if the supplier chooses a contract (t, q), the retailer will generate a profit $v(q) - t$. The retailer now needs to design a menu of contract to maximize her expected profit.

(a) Formulate the retailer's contract design problem by assuming there is no information asymmetry. Then solve the problem to obtain the first-best menu.

(b) Formulate the retailer's contract design problem for finding the second best menu.

(c) Solve for the second best menu.

(d) Demonstrate "monotonicity," "efficiency at top," and "no rent at bottom." For θ_H and θ_L, which is "top" and which is "bottom"?

5. Consider a set of consumers whose types θ lie in an interval $[0, 1]$ uniformly. Each of these consumers are considering buying a product of two versions with quality levels q_1 and q_2, where $0 < q_1 < q_2$. A type-θ consumer's utility is $\theta q - p$ if he buys the version of quality q by paying p to the seller. A consumer can either buy version 1, buy version 2, or buy nothing (which gives him a zero utility). He makes the decision to maximize his utility. Let the prices of the two versions be p_1 and p_2, respectively, where $p_1 < p_2$. We assume that $q_1 > p_1$ and $q_2 > p_2$ so that at least the highest-type consumer is willing to buy something. We further assume that

$$\frac{p_2 - p_1}{q_2 - q_1} > \frac{p_1}{q_1}. \tag{1}$$

(a) For a type-θ consumer, under what condition will he prefers buying version 1 to buying nothing? Is this some kind of IR constraint?

(b) For a type-θ consumer, under what condition will he prefers buying version 2 to version 1? Is this some kind of IC constraint?

(c) What does the assumption in (1) imply on market segmentation?

(d) Formulate the seller's problem of choosing p_1 and p_2 to maximize her total profit.

(e) Solve the seller's problem to find the optimal p_1 and p_2. How do q_1 and q_2 affect the two optimal prices?

6. In this problem, we consider a three-type monopoly pricing problem. Suppose now $\theta \in \{\theta_1, \theta_2, \theta_3\}$, where $\theta_1 < \theta_2 < \theta_3$. The seller believes that $\Pr(\theta = \theta_i) = \beta_i$, where $\beta_1 + \beta_2 + \beta_3 = 1$. Assume that it is the seller's best interest to do business with all three types of consumers.

   (a) Formulate the seller's contract design problem. How many IC and IR constraints do you have?

   (b) Show that $q_1^* \leq q_2^* \leq q_3^*$, where q_i^* is the second-best consumption level of the type-i consumer.

   (c) Show that (IR-3) and (IR-2) are both redundant, where (IR-i) is to ensure the type-i consumer to participate.

   (d) Show that (IC-$(3, 1)$) is redundant, where (IC-(i, j)) is to prevent the type-i consumer to pretend to be the type-j consumer.

(e) Suppose someone tells you that four IC constraints (including IC-$(3, 1)$) must be satisfied by the second-best menu. Intuitively guess which four can be removed due to this reason.

(f) Suppose those four IC constraints are indeed removed. For the remaining three constraints (one IR, two IC), show that all of them must be binding at the optimal solution.