# Information Economics

# The Signaling Theory

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# Road map

- **Introduction**.
- Signaling with a discrete action space.
- Signaling with a continuous action space.

# Signaling

- We have studied the **screening** problem.
  - The **agent** has hidden information.
- Today we will study the **signaling** problem.
  - The **principal** has hidden information.
- Both screening and signaling are adverse selection issues.

# Origin of the signaling theory

- Akerlof (1970) studies the market of **used cars**.
  - The owner of a used car knows the **quality** of the car.
  - Potential buyers, however, do not know it.
  - The quality is hidden information observed only by the principal (seller).
- What is the issue?
  - Buyers do not want to buy "lemons".
  - They only pay a price for a used car that is "**around average**".
  - Owners of **bad** used cars are happy for selling their used cars.
  - Owners of **good** ones do not sell theirs.
  - Days after days... there are only bad cars on the market.
  - The "expected quality" and "average quality" become lower and lower.
- **Information asymmetry** causes **inefficiency**.
  - In screening problems, information asymmetry protects agents.
  - In signaling problems, information asymmetry **hurts everyone**.
- That is why we need platforms that suggest prices for used cars.

# Origin of the signaling theory

- Spence (1973) studies the market of **labors**.
  - One knows her **ability** (productivity) while potential employers do not.
  - The "quality" of the worker is hidden.
  - Firms only pay a wage for "**around average**" workers.
  - **Low**-productivity workers are happy. **High**-productivity ones are sad.
  - Productive workers leave the market (e.g., go abroad). Wages decrease.
- What should we do? No platform can suggest wages for individuals!
- That is why we get **high education** (or study in good schools).
  - It is not very costly for a high-productivity person to get a higher degree.
  - It is **more costly** for a low-productivity one to get it.
  - By getting a higher degree (e.g., a master), high-productivity people **differentiate** themselves from low-productivity ones.
  - Getting a higher degree is **sending a signal**.
- This will happen (as an equilibrium) even if education itself **does not** enhance productivity!

# Signaling

- Signaling is for the principal to send a message to the agent to **signal the hidden information**.
  - Sending a message requires an **action** (e.g., getting a degree).
- For signaling to be effective, different types of principal should take different actions.
  - It must be **too costly** for a type to take a certain action.
- Other examples:
  - A manufacturer offers a **warranty** policy to signal the product reliability.
  - A firm sets a high **price** to signal the product quality.
  - "Full **refund** if not tasty".

# Signaling games

- How to model a signaling game between a principal and an agent?
  - The principal has a **hidden type**.
  - The agent cannot observe the type and thus have a **prior belief** on the principal's type.
  - The principal chooses an **action** that is observable.
  - The agent then forms a **posterior belief** on the type.
  - Based on the posterior belief, the agent **responds** to the principal.
- The principal takes the action to **alter** the agent's belief.
- An example:
  - A firm makes and sells a product with **hidden reliability** to consumers.
  - Consumers have a prior belief on the reliability.
  - The firm chooses between **offering a warranty or not**.
  - By observing the policy, the consumer **updates his belief** and make the purchasing decision accordingly.
- We need to model belief updating by **the Bayes' theorem**.

# Road map

- Introduction.
- **Signaling with a discrete action space.**
- Signaling with a continuous action space.

# The first example

- A firm makes and sells a product with **hidden reliability** $r \in (0,1)$.
  - $r$ is the probability for the product to be functional.
- If a consumer buys the product at price $t$:
  - If the product works, his utility is $\theta - t$.
  - If the product fails, his utility is $-t$.
- The firm may offer a **warranty** plan and repair a broken product.
  - The firm pays the repairing cost $k > 0$.
  - The consumer's utility is $\eta \in (0, \theta)$.
- The price is fixed (exogenous).
- Suppose $w = 1$ if a warranty is offered and 0 otherwise.
- Expected utilities:
  - The firm's expected utility is $u_F = t - (1-r)kw$.
  - The consumer's expected utility is $u_C = r\theta + (1-r)\eta w - t$.
- The consumer buys the product if and only if $u_C \geq 0$.
- The firm chooses whether to offer the warranty accordingly.

# The first example: no signaling

- Suppose $r \in \{r_H, r_L\}$: The product may be reliable or unreliable.
  - $0 < r_L < r_H < 1$.
- Under complete information, the decisions are simple.
  - The firm's expected utility is $u_F = t - (1 - r_i)kw$.
  - The consumer's expected utility is $u_C = r_i\theta + (1 - r_i)\eta w - t$.
- Under incomplete information, they may make decision according to the **expected reliability**:
  - Let $\beta = \Pr(r = r_L) = 1 - \Pr(r = r_H)$ be the consumer's **prior belief**.
  - The expected reliability is $\bar{r} = \beta r_L + (1 - \beta)r_H$.
  - The firm's expected utility is $u_F = t - (1 - r_i)kw$.
  - The consumer's expected utility is $u_C = \bar{r}\theta + (1 - \bar{r})\eta w - t$.
- But wait! The **unreliable** firm will tend to offer **no warranty**.
  - Because $(1 - r_L)k$ is high.
  - This forms the basis of **signaling**.

# The first example: signaling

- Below we will work with the following parameters:
  - $r_L = 0.2$ and $r_H = 0.8$.
  - $\theta = 20$ and $\eta = 5$.
  - $t = 11$ and $k = 15$.
- Payoff matrices (though players make decisions sequentially):

Consumer

| Firm | Buy | Not |
|---|---|---|
| $w = 1$ | $8, 6$ | $0, 0$ |
| $w = 0$ | $11, 5$ | $0, 0$ |

(Product is reliable)

Consumer

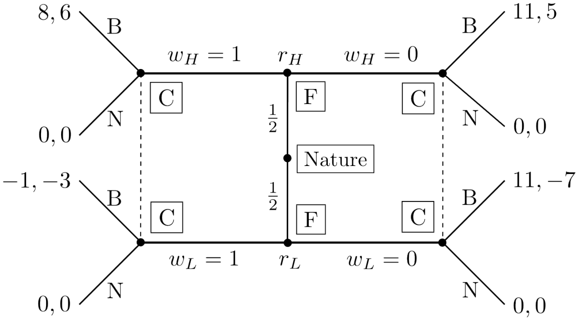
| Firm | Buy | Not |
|---|---|---|
| $w = 1$ | $-1, -3$ | $0, 0$ |
| $w = 0$ | $11, -7$ | $0, 0$ |

(Product is unreliable)

- The issue is: The consumer does not know **which matrix** he is facing!
- The reliable firm tries to convince the consumer that it is the first one.

# Game tree

- We express this **game with incomplete information** by the following game tree:
  - F and C: players.
  - Nature: a fictitious player that draws the type randomly.
  - Let $\beta = \frac{1}{2}$ be the prior belief.

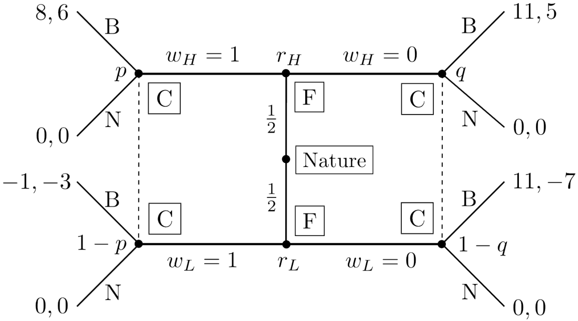


# Concept of equilibrium

- What is a (pure-strategy) **equilibrium** in a signaling game?
- Decisions:
  - The "two" firms' actions: $(w_H, w_L)$, $w_i \in \{0, 1\}$.
  - The consumer's strategy: $(a_1, a_0)$, $a_j \in \{B, N\}$.
- Posterior beliefs:
  - Let $p = \Pr(r_H | w = 1)$ be the posterior belief upon observing a warranty.
  - Let $q = \Pr(r_H | w = 0)$ be the posterior belief upon observing no warranty.
- An equilibrium is a strategy-belief **profile** $((w_H, w_L), (a_1, a_0), (p, q))$:
  - No firm wants to deviate based on the consumer's posterior belief.
  - The consumer does not deviate based on his posterior belief.
  - The beliefs are updated according to the firms' actions by the Bayes' rule.
- It is extremely hard to "search for" an equilibrium. It is easier to "**check**" whether a given profile is one.
- We start from the firms' actions:[1]
  - Can $(1, 0)$ be part of an equilibrium? How about $(0, 1)$, $(1, 1)$, and $(0, 0)$?

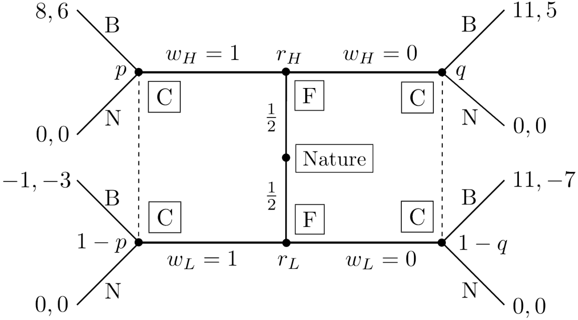
[1] It is typical to start from the principal's actions.

# Warranty for the reliable product only



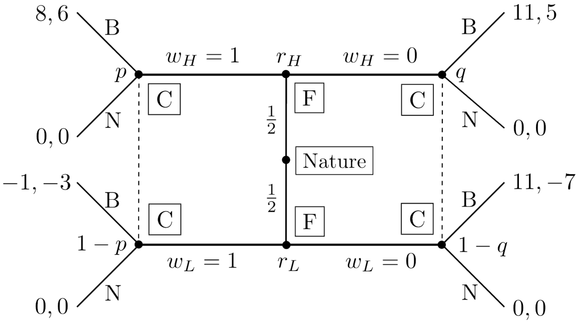

- We start from $((1,0),(a_1,a_0),(p,q))$.
- Bayesian updating: $p = 1$, $q = 0$: $((1,0),(a_1,a_0),(1,0))$.
- Consumer $((1,0),(B,N),(1,0))$.
- No firm wants to deviate.

# Warranty for the unreliable product only



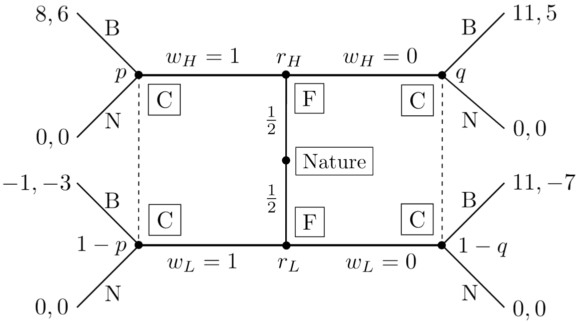

- We start from $((0,1),(a_1,a_0),(p,q))$.
- Bayesian updating: $p = 0$, $q = 1$: $((0,1),(a_1,a_0),(0,1))$.
- Consumer: $((0,1),(N,B),(0,1))$.
- But now the unreliable firm deviates to $w_L = 0$!

# Both offering warranties




- We start from $((1,1),(a_1,a_0),(p,q))$.
- Bayesian updating: $p = \frac{1}{2}$, $q \in [0,1]$: $((1,1),(a_1,a_0),(\frac{1}{2},[0,1]))$.
- Consumer: $((1,1),(B,\{B,N\}),(\frac{1}{2},[0,1]))$.
- If $a_0 = B$, no firm offers a warranty: $((1,1),(B,N),(\frac{1}{2},[0,1]))$.
- But now the unreliable firm deviates to $w_L = 0$!

# Both offering no warranty




- We start from $((0,0),(a_1,a_0),(p,q))$.
- Bayesian updating: $p \in [0,1]$, $q = \frac{1}{2}$: $((0,0),(a_1,a_0),([0,1],\frac{1}{2}))$.
- Consumer: $((0,0),(B,N),([\frac{1}{3},1],\frac{1}{2}))$, or $((0,0),(N,N),([0,\frac{1}{3}],\frac{1}{2}))$.
- For the former, the reliable firm deviates to $w_H = 1$. The latter is a pooling equilibrium.

# Interpretations

- There are **pooling**, **separating**, and **semi-separating** equilibria:
  - In a pooling equilibrium, all types take the same action.
  - In a separating equilibrium, different types take different actions.
  - In a semi-separating one, some but not all types take the same action.
- In this example, there are two (sets of) equilibria:
  - A separating equilibrium $((1,0),(B,N),(1,0))$.
  - A pooling equilibrium $((0,0),(N,N),([0,\frac{1}{3}],\frac{1}{2}))$.
- What does that mean?

# Interpretations

- The separating equilibrium is $((1,0),(B,N),(1,0))$:
  - The reliable product is sold with a warranty.
  - The unreliable product, offered with no warranty, is not sold.
  - The reliable firm **successfully signals** her reliability.
  - The system becomes more efficient.
  - Because it is too costly for the unreliable firm to do the same thing.
- The pooling equilibrium is $((0,0),(N,N),([0,\frac{1}{3}],\frac{1}{2}))$.
  - Both firms do not offer a warranty.
  - The consumer cannot update his belief.
  - The consumer does not buy the product.
- In this (and most) signaling game, there are **multiple** equilibria.

# Road map

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# The second example

- A manufacturer sells a product of hidden reliability $r \in \{r_L, r_H\}$.
  - The consumer's prior belief on $r$ is $\Pr(r = r_L) = \beta = 1 - \Pr(r = r_H)$.
- The manufacturer chooses a **price** $t \in \mathbb{R}$ and a **warranty protection probability** $w \in [0, 1]$.
- By selling the product, the type-$i$ manufacturer's expected utility is

$$u_i^M(t, w) = t - (1 - r_i)wk.$$

  - $k$ is the cost of fixing a broken product.
- By buying the product with $r$ as the expected reliability, the consumer's expected utility is

$$u^C = r\theta + (1 - r)\eta w - t.$$

  - $\theta$ is the utility of using a functional product.
  - $\eta$ is the utility of using a fixed product. $k > \eta$ and $\theta > \eta$.

# First best

- Assume that $r$ is **public**. Consider the type-$i$ manufacturer's first-best offer $(t_i^{FB}, w_i^{FB})$ with reliability $r_i$.
- The manufacturer's problem is

$$\begin{aligned} \max_{t \in \mathbb{R}, w \in [0,1]} \quad & t - (1 - r_i)wk \\ \text{s.t.} \quad & r_i\theta + (1 - r_i)\eta w - t \geq 0. \end{aligned}$$

- This reduces to

$$\max_{w \in [0,1]} r_i\theta + (1 - r_i)(\eta - k)w.$$

- As $\eta < k$, we have $w^{FB} = 0$ and thus $t^{FB} = r_i\theta$.
- Both types of manufacturers offer **no warranty**.

## Second best

- Assume that $r$ is private. Let's look for a **separating equilibrium**.
- Let's guess: The unreliable manufacturer chooses its **first-best** offer $(t_L^*, w_L^*) = (r_L\theta, 0)$.
- Let's try to find the reliable manufacturer's offer $(t_H^*, w_H^*)$ in this case.
- The reliable manufacturer's problem is

$$
\begin{aligned}
\max_{t_H \in \mathbb{R}, w_H \in [0,1]} \quad & t_H - (1 - r_H) w_H k \\
\text{s.t.} \quad & r_H \theta + (1 - r_H) \eta w_H - t_H \geq 0 && \text{(IR)} \\
& t_L^* - (1 - r_L) w_L^* k \geq t_H - (1 - r_L) w_H k && \text{(IC-L)} \\
& t_H - (1 - r_H) w_H k \geq t_L^* - (1 - r_H) w_L^* k. && \text{(IC-H)}
\end{aligned}
$$

# Solving for the second best

- By replacing $t_L^*$ and $w_L^*$ by $r_L\theta$ and 0, the problem reduces to

$$\begin{aligned}\max_{t_H\in\mathbb{R},w_H\in[0,1]} \quad & t_H-(1-r_H)w_Hk \\ \text{s.t.} \quad & r_H\theta+(1-r_H)\eta w_H-t_H\geq 0 && \text{(IR)} \\ & r_L\theta\geq t_H-(1-r_L)w_Hk && \text{(IC-L)} \\ & t_H-(1-r_H)w_Hk\geq r_L\theta. && \text{(IC-H)}\end{aligned}$$

- Let's ignore (IC-H) for a while.

# Solving for the second best

- By replacing $t_L^*$ and $w_L^*$ by $r_L\theta$ and 0, the problem reduces to

$$
\begin{aligned}
\max_{t_H \in \mathbb{R}, w_H \in [0,1]} \quad & t_H - (1 - r_H) w_H k \\
\text{s.t.} \quad & r_H\theta + (1 - r_H)\eta w_H - t_H \geq 0 \quad \text{(IR)} \\
& r_L\theta \geq t_H - (1 - r_L) w_H k. \quad \text{(IC-L)}
\end{aligned}
$$

- Suppose that (IC-L) is not binding, then (IR) is binding, and the problem reduces to

$$\max_{w_H \in [0,1]} \; r_H\theta + (1 - r_H)(\eta - k) w_H,$$

and the optimal solution is $w_H = 0$ and $t_H = r_H\theta$. This violates (IC-L), so we know (IC-L) must be binding.

# Solving for the second best

- The problem reduces to

$$\max_{w_H \in [0,1]} \quad r_L\theta + (r_H - r_L)w_H k$$
$$\text{s.t.} \quad (r_H - r_L)\theta + \Big[(1 - r_H)\eta - (1 - r_L)k\Big]w_H \geq 0. \quad \text{(IR)}$$

- To solve this problem, note that:
  - $r_H > r_L$ and $k > \eta$ implies $(1 - r_H)\eta - (1 - r_L)k < 0$.
  - The objective function is increasing in $w_H$.
- Collectively, we have

$$w_H^* = \min\left\{1, \frac{(r_H - r_L)\theta}{(1 - r_L)k - (1 - r_H)\eta}\right\} \quad \text{and} \quad t_H^* = r_L\theta + (1 - r_L)w_H^* k.$$

- It is straightforward to verify that (IC-H) is satisfied.

# Interpretations

- In a separating equilibrium, the consumer may **tell** whether the manufacturer is reliable or not.
- The unreliable manufacturer chooses its **first-best offer**.
  - Its type will be revealed anyway.
  - It should choose the "most efficient" offer, i.e., the first-best one.
- The reliable manufacturer **upward distorts** its warranty protection probability.
  - $w_H^* > w_H^{FB} = 0$.
  - To discourage the unreliable one from mimicking itself.