1. Consider the signaling game of warranty offering illustrated in Figure 1. In this game, there are three possible realizations of reliability: $r_H = 0.8$, $r_M = 0.6$, and $r_L = 0.2$. The prior belief is $\Pr(r = r_i) = \frac{1}{3}$ for $i \in \{H, M, L\}$. Let (w_H, w_M, w_L) be the firm's strategy.

   (a) Is $(w_H, w_M, w_L) = (1, 1, 0)$ part of a possible equilibrium? If so, what is the corresponding posterior belief and the consumer's strategy?

   (b) How about $(w_H, w_M, w_L) = (1, 0, 0)$?

   (c) How about $(w_H, w_M, w_L) = (1, 1, 1)$?

   (d) How about $(w_H, w_M, w_L) = (0, 0, 0)$?

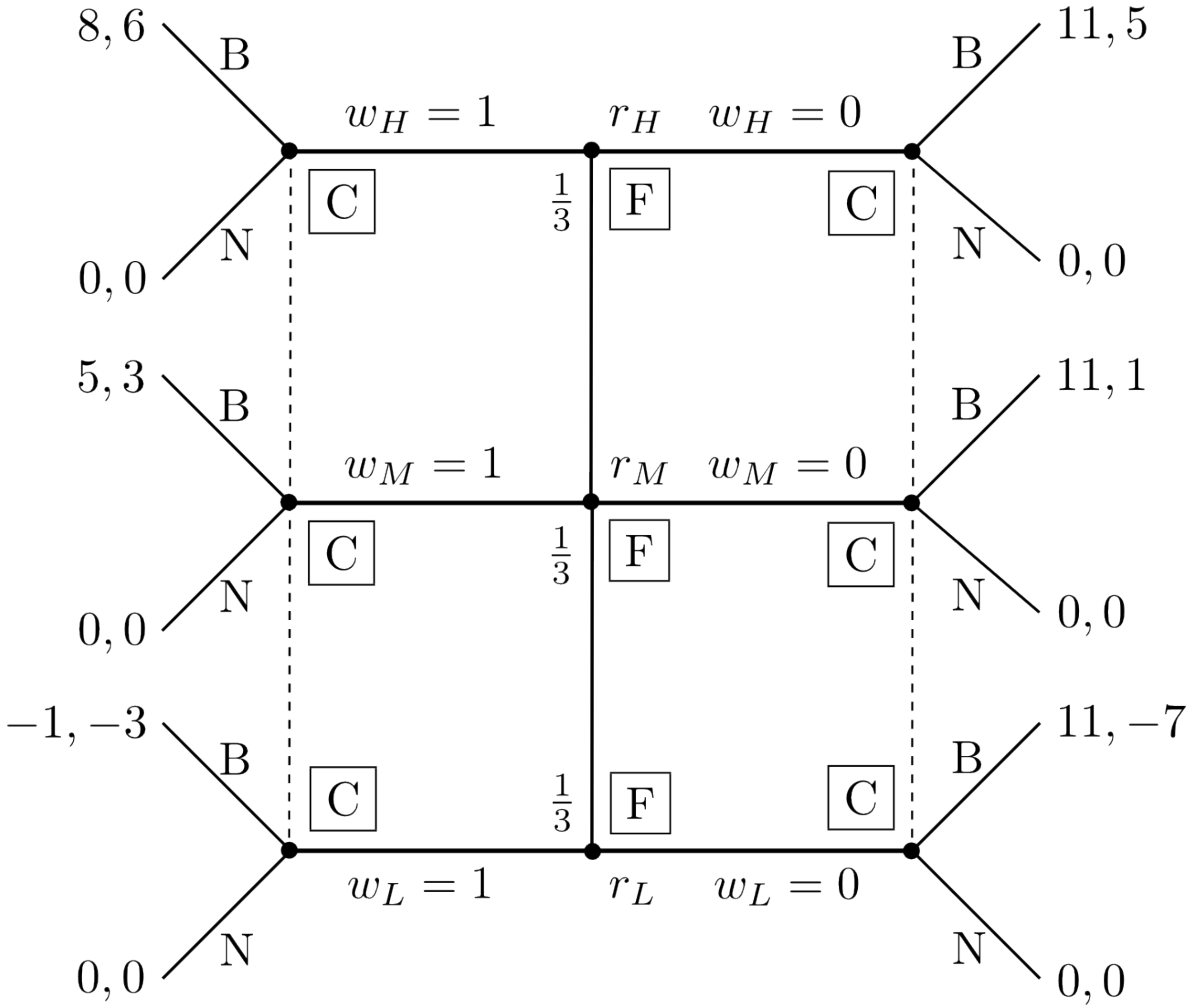

Figure 1: Warranty offering with three quality levels

2. For the warranty game discussed in class, consider the following mixed strategy in which $\Pr(w_H = 1) = 1$ and $\Pr(w_L = 1) = \frac{1}{2}$.

   (a) What is the posterior belief?

   (b) Is the mixed strategy part of a possible equilibrium?

3. Consider the signaling game of warranty offering illustrated in Figure 2. In this game, the prior belief is that

$$\Pr(r = r_H) = \lambda = 1 - \Pr(r = r_L).$$

(a) Suppose that $\lambda \geq \frac{7}{12}$, what are the possible equilibria?

(b) Suppose that $\lambda \leq \frac{1}{3}$, what are the possible equilibria?

(c) Suppose that $\frac{1}{3} \leq \lambda \leq \frac{7}{12}$, what are the possible equilibria?

(d) Does a lower λ give the reliable firm a higher incentive to signal its reliability by offering a warranty?

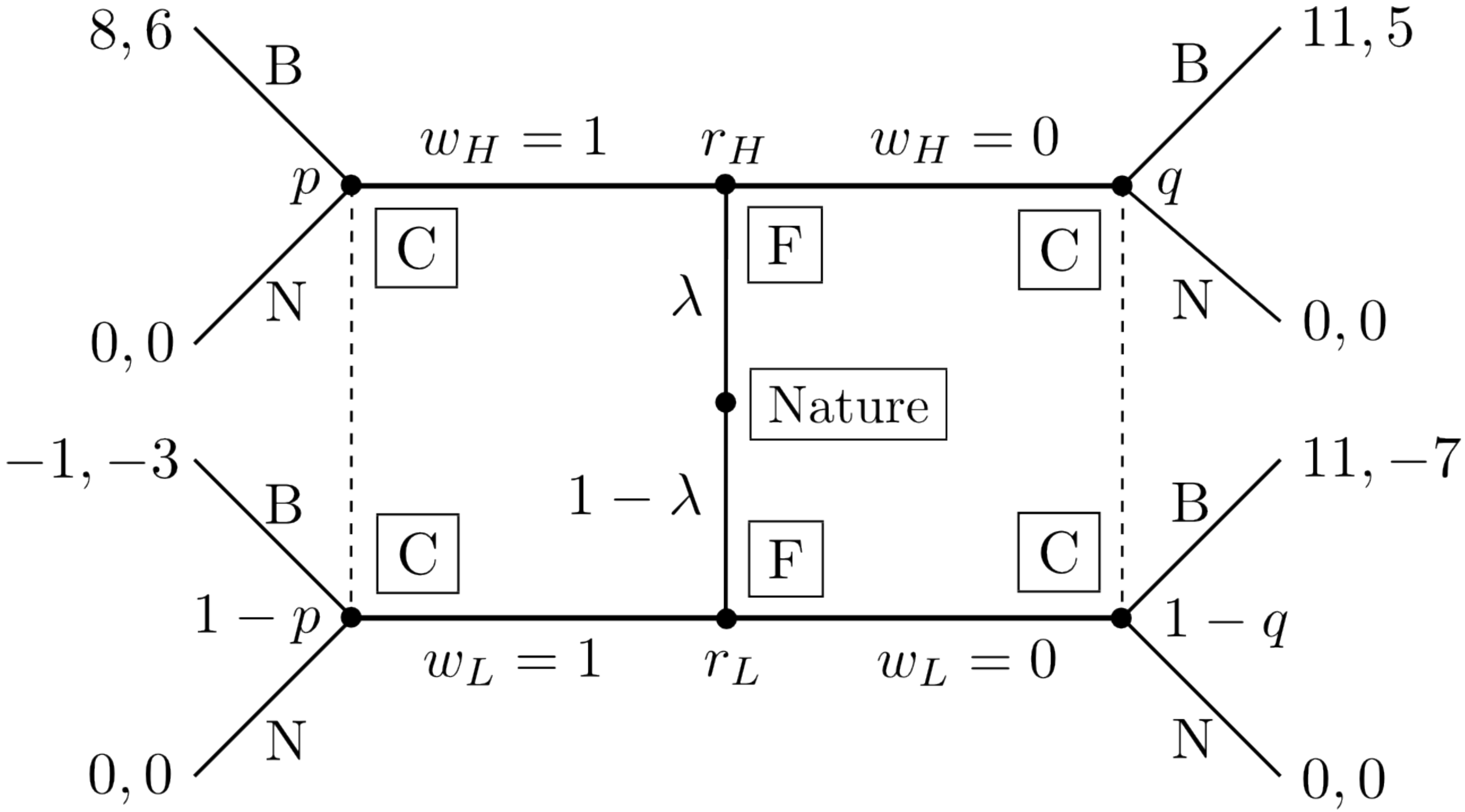

Figure 2: Warranty offering with a general prior belief

4. A seller is going to sell a product of quality q at price p. The quality $q \in \{q_L, q_H\}$ is privately observed by the seller and is hidden to the consumers. The consumers believe that $\Pr(q = q_L) = \beta = 1 - \Pr(q = q_H)$ for some $\beta \in (0, 1)$. Consumers' willingness-to-pay θ is uniformly distributed between 0 and 1. A type-θ consumer buys the product if his utility $\theta\tilde{q} - p \geq 0$, where $\tilde{q}$ is the quality level in his belief. The unit production costs are c_H for the high-quality quality and c_L for the low-quality one. We normalize c_L to 0. It is publicly known that $q_H > q_L > 2c_H > 0$.

   (a) Suppose that there is no information asymmetry, find the type-i seller's first-best price p_i^{FB}, $i \in \{L, H\}$.

(b) Suppose that there is information asymmetry, consider a separating equilibrium in which the high-quality seller's price p_H is different from that of the low-quality seller p_L. Suppose that the low-quality seller sets p_L to the first-best level. Convince yourself that the high-quality seller's optimization problem is

$$\begin{aligned} \max_{p_H} \quad & \left(1 - \frac{p_H}{q_H}\right)(p_H - c_H) \\ \text{s.t.} \quad & \left(1 - \frac{p_L^{FB}}{q_L}\right)(p_L^{FB} - c_L) \geq \left(1 - \frac{p_H}{q_H}\right)(p_H - c_L) \\ & \left(1 - \frac{p_H}{q_H}\right)(p_H - c_H) \geq \left(1 - \frac{p_L^{FB}}{q_L}\right)(p_L^{FB} - c_H). \end{aligned}$$

Explain the meanings of the objective function and constraints in words.

(c) Show that the above program is feasible.

(d) Show that the high-quality seller's first-best price p_H^{FB} always satisfies the second constraint.

(e) If you try to plug in p_H^{FB} into the first constraint, the first constraint will be satisfied if and only if $q_L q_H \geq q_H^2 - c_H^2$. When will this condition be satisfied? What if $c_H = 0$? Intuitively explain why.

(f) Suppose that $q_L q_H < q_H^2 - c_H^2$, show how the first constraint will impose restrictions on p_H for separation.