Reliability Signaling through Revenue Sharing for Medical Treatments

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Road map

- **Background and motivation**.
- Model.
- Analysis.
- Conclusions.

Radiation treatment for cancers

- More than 50% of cancer patients (in Taiwan) get **radiation treatment**.[1]
- Radiation equipment (e.g., linear accelerators) is critical for radiation treatment.
  - IMRT: Intensity-moderated radiation treatment.
- The typical process of radiation treatments:
  - Ten to thirty minutes per day.
  - Once per day, five days per week.

[1] This lecture is based on a working paper written by the instructor and the other authors.

Radiation equipment purchasing

- Traditionally, an equipment vendor **sells** linear accelerators to hospitals at a single price.
- Now it is also common for a vendor to give accelerators to hospitals "for free."
  - In exchange for per-treatment payments.
  - The vendor is adopting a **revenue-sharing** contract.
- Why?
  - Does the vendor earn more with revenue-sharing?
  - If so, why is a hospital willing to accept?

Radiation equipment purchasing

- Typical reasons:
  - A hospital's annual budget may be limited.
  - A salesperson may prefer steady sales performance.
- Beside these **significant** factors, is there any **insignificant** factor?
- Research questions:
  - What are the (insignificant) factors that affecting the contract format between a hospital and a vendor?
  - If there is one, why?

Data

- We collect data from 27 hospitals which have acquired at least one machine R for cancer diagnoses.

Variable	Meaning
Name	The name of hospitals
Private	The hospital is held by a private organization (1) or is held by the government (0)
Regional	The level of the hospital is regional (1) or teaching hospital (0)
Location	The location of the hospital is at the north (1), west (2), south (3), or east (4) of Taiwan
Bed	The number of beds in a hospital
Buy	The number of machines rent by the hospital
Rent	The number of machines bought by the hospital
RentPercentage	$Rent/(Buy + Rent)$

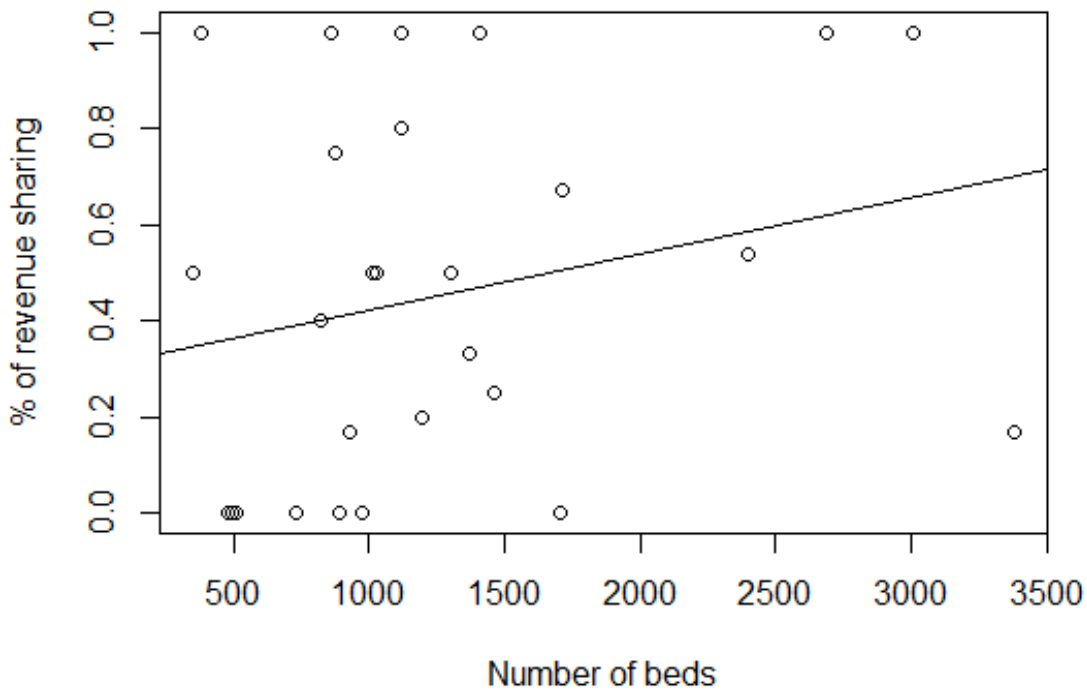
Data

- The data:

Name	*Private*	*Regional*	*Location*	*Bed*	*Buy*	*Rent*
H1	0	0	1	1,712	1	2
H2	1	0	2	1,305	1	1
H3	1	1	1	732	3	0
H4	0	0	2	1,464	3	1
H5	0	0	1	3,010	0	1
			...			
H25	0	0	1	2,400	6	7
H26	0	1	2	510	1	0
H27	0	1	1	1,120	0	3

Empirical observations about hospital size

- Average *RentPercentage*: 54.4% for teaching and 32.5% for regional.
- Correlation coefficient between *RentPercentage* and *Bed*: 0.2398.



- Large hospitals (slightly) prefer renting more than small hospitals do.

Why renting?

- Hospital budget and salesperson's intention may indeed be reasons.
- There must be some other reasons.
- According to the data, it seems that the **management type** (public or private) matters.
  - Average *RentPercentage*: 51.8% for public and 42.3% for private.
  - **Public** hospitals prefer **renting** more than private ones do.
  - Public $\approx$ non-profit; private $\approx$ for-profit.
- Is it true that a public hospital has a reason to prefer renting more than a private one does (given that all other conditions are the same)?
- If so, what difference between these two types leads to the result?
  - In general, private hospitals care more about **profit maximization**.
  - Is this a reason? If so, why?

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Model

- A vendor (she) contracts with a hospital (he) for a kind of medical equipment required for a certain treatment.
- The medical treatment requires reservation.
  - The maximum number of patients that can be served in a period is K.
  - K is called the **capacity** of the machine.
- The probability that the machine is functional is r.
  - $r \in \{r_L, r_H\}$ is the vendor's private information. $0 < r_L < r_H < 1$.
  - For the hospital, the prior belief on r is $\Pr(r = r_L) = \beta = 1 - \Pr(r = r_H)$.
  - r is called the **reliability** of the machine.
- Once the machine is down, affected treatments will be postponed but not canceled.
  - The **effective capacity** is rK.
  - There is no "lost sales."

Non-profit and for-profit hospitals

- The hospital may be **non-profit** (public) or **for-profit** (private).
- For a for-profit hospital, the treatment price p is **endogenously chosen** to maximize its profit.
  - The demand for the treatment is $D(p) = a - bp$.
  - $D(p)$ may be above or below rK. The treatment volume is

$$\min\{D(p), rK\}.$$

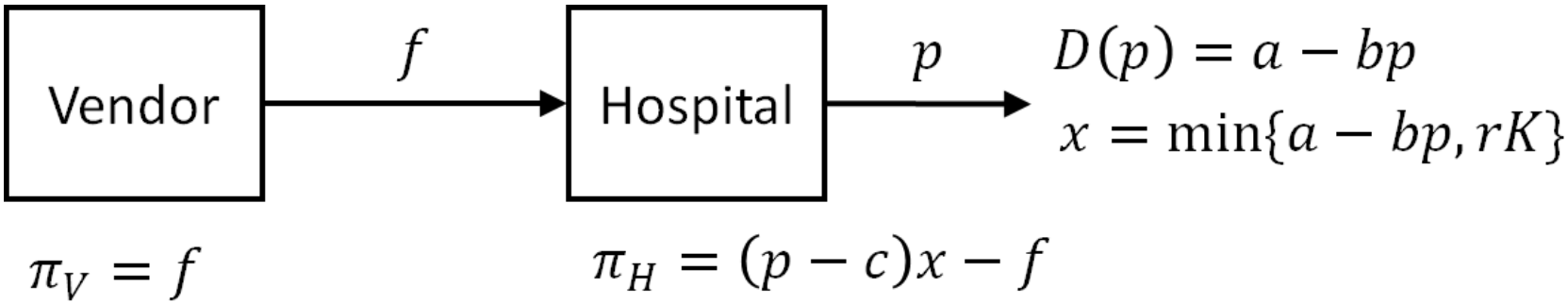
- For a non-profit hospital, the price p_0 per treatment is **exogenous**.
  - We assume that $D(p_0) = a - bp_0 > rK$ in this case.
- The unit treatment cost is c.

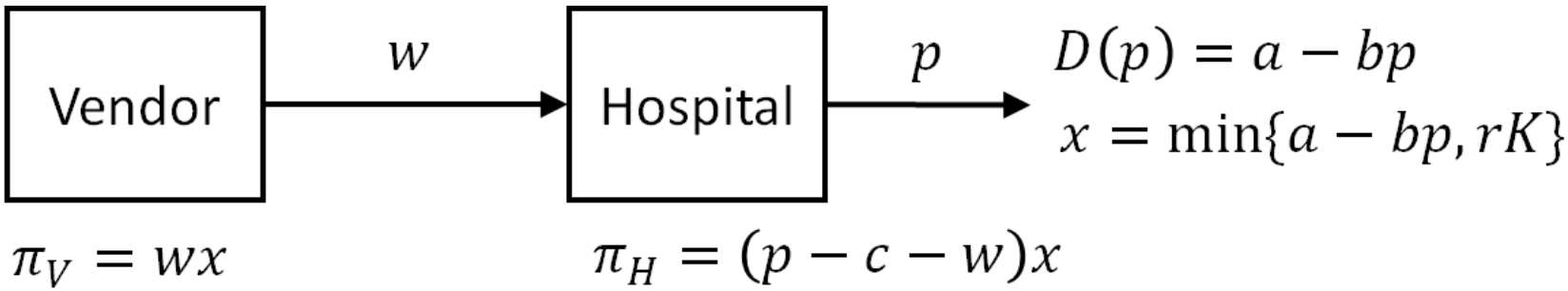
Contracting

- The vendor offers the hospital two options:[2]
  - **Fixed-fee contract**: The machine is sold at a fixed fee f.
  - **Revenue-sharing contract**: The hospital pays w per treatment.
- In either case, the vendor chooses f or w for profit maximization.

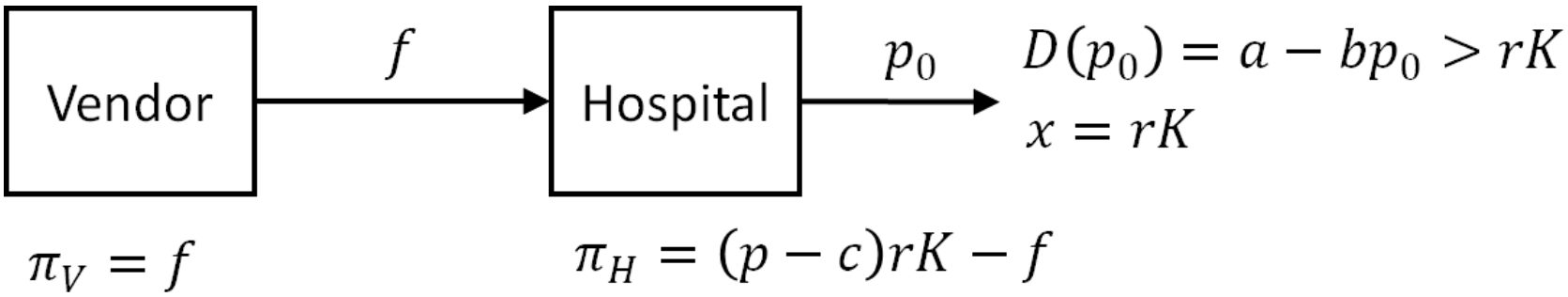
[2] In general, the contract may be a mixed one including both a fixed fee and a per-treatment fee. Here we discuss pure contracts only.

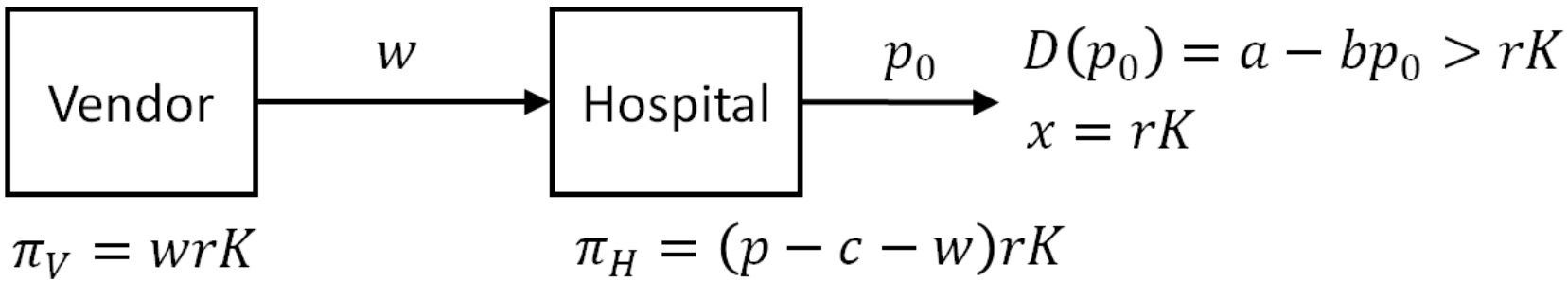
Model: for-profit hospital





Model: non-profit hospital





Sequence of events

- Sequence of events:
  - The vendor privately observes $r \in \{r_L, r_H\}$.
  - The vendor offers one of the two contracts.
  - The hospital updates his belief on r by observing the offer.
  - The hospital accepts or rejects the offer. Payments are made accordingly.
- Is it possible for the reliable vendor to signal her reliability?

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- Background and motivation.
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- **Analysis**.
  - **Non-profit hospitals**.
  - For-profit hospitals.
- Conclusions.

Non-profit hospitals: public reliability

- Suppose that r is common knowledge first.
- A public hospital does not make the pricing decision.
  - Both p_0 and c are fixed.
  - The demand $D(p_0)$ is above the capacity rK.
  - His profit is

$$\begin{cases} (p_0 - c)rK - f & \text{with a fixed fee } f \\ (p_0 - c - w)rK & \text{with a per-treatment payment } w \end{cases}.$$

- For the vendor:
  - The optimal fixed fee is $f = (p_0 - c)rK$.
  - The optimal per-treatment payment is $w = p_0 - c$.
  - She earns $(p_0 - c)rK$ anyway.

Non-profit hospitals: private reliability

- When machine reliability r is **hidden**, the hospital's willingness-to-pay depends on his belief on r.
  - May the reliable vendor differentiate itself from the unreliable one?
- There are four possible combinations of contract offering: (F, F), (F, R), (R, F), and (R, R).
  - E.g., (F, R) means that the unreliable vendor offers a fixed fee whereas the reliable vendor offers a per-treatment fee.
- Note that separation is impossible when the two types of vendors offer the same type of contract.
  - E.g., under (F, F), the unreliable vendor may always mimic the reliable one by offering the same fixed fee.
  - **Price alone cannot** be a signaling device.
- May (F, R) or (R, F) exist as a separating equilibrium?

Non-profit hospitals: private reliability

Proposition 1

When the vendor with hidden reliability sells to a non-profit hospital, the separating equilibrium (F, R) always exists. In this equilibrium, we have

$$f_L^N = (p_0 - c) r_L K \quad \text{and} \quad w_H^N = p_0 - c,$$

where f_L^N and w_H^N are the fixed fee charged by the unreliable vendor and the per-treatment payment charged by the reliable vendor, respectively.

- Each firm chooses one contract format and offers her **first-best** price.
- **Contract format** is a useful signaling device.
- One may signal her high reliability by offering **revenue sharing**.

Proof of Proposition 1

- For the unreliable vendor:
  - Her first-best fixed-fee contract is $f_L^N = (p_0 - c)r_L K$.
  - Mimicking the reliable one by switching to $w_H^N = p_0 - c$ results in the same expected profit $(p_0 - c)r_L K$.
- For the reliable vendor:
  - Her first-best revenue-sharing contract is $w_H^N = p_0 - c$.
  - Mimicking the unreliable vendor results in a lower profit f_L^N as $r_L < r_H$.
- No one wants to unilaterally deviate.

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For-profit hospitals: public reliability

- Upon accepting a fixed fee f, the hospital solves

$$\max_p \ (p-c)\min\{a-bp, rK\} - f.$$

The optimal treatment price is

$$\begin{cases} \frac{a+bc}{2b} & \text{if } rK \geq \frac{a-bc}{2} \\ \frac{a-rK}{b} & \text{otherwise} \end{cases}.$$

- For the vendor, the optimal fixed fee (and his expected profit) is

$$f_{FB}^F = \begin{cases} \frac{(a-bc)^2}{4b} & \text{if } rK \geq \frac{a-bc}{2} \\ \frac{(a-bc-rK)rK}{b} & \text{otherwise} \end{cases}.$$

For-profit hospitals: public reliability

- Upon accepting a per-treatment payment w, the hospital solves

$$\max_p \ (p - c - w) \min\{a - bp, rK\}.$$

  The optimal treatment price is

$$\begin{cases} \frac{a+bc+bw}{2b} & \text{if } rK \geq \frac{a-bc-bw}{2} \\ \frac{a-rK}{b} & \text{otherwise} \end{cases}.$$

- The equilibrium price is **higher** than that with a fixed fee.
  - Double marginalization.

For-profit hospitals: public reliability

- The vendor solves

$$\max_{w} \; w \min\{a - bp, rK\}.$$

  The optimal per-treatment fee is

$$\begin{cases} \frac{a-bc}{2b} & \text{if } rK \geq \frac{a-bc}{4} \\ \frac{a-bc-2rK}{b} & \text{otherwise} \end{cases}.$$

- The vendor's expected profit is

$$\begin{cases} \frac{(a-bc)^2}{8b} & \text{if } rK \geq \frac{a-bc}{4} \\ \frac{(a-bc-2rK)rK}{b} & \text{otherwise} \end{cases}.$$

- When r is public, a vendor always prefer a fixed-fee contract.
  - Using a per-treatment fee **cannot** extract all the surplus.
  - The revenue-sharing contract is **inefficient** due to double marginalization.

For-profit hospitals: private reliability

- When machine reliability is **hidden**:
  - If the reliable vendor chooses to offer the fixed-fee contract, she will **be mimicked** by the unreliable vendor.
  - To convince the hospital of her high reliability, the reliable vendor can only provide the revenue-sharing contract.
  - Unfortunately, the revenue-sharing contract is inefficient when the hospital is for-profit.
- When is the benefit of signaling large enough to cover the detriment of double marginalization?
- Is signaling still possible?

For-profit hospitals: private reliability

Proposition 2

Suppose that the vendor with hidden reliability sells to a for-profit hospital. A separating equilibrium does not exist if

$$a - bc < \min\left\{(4 + 2\sqrt{2})r_L K, (r_H + r_L)K\right\}.$$

- Signaling is still possible.
- However, it is impossible if:
  - The profit potential $a - bc$ is small.
  - The unreliable vendor's effective reliability $r_L K$ is high.

Proof of Proposition 2

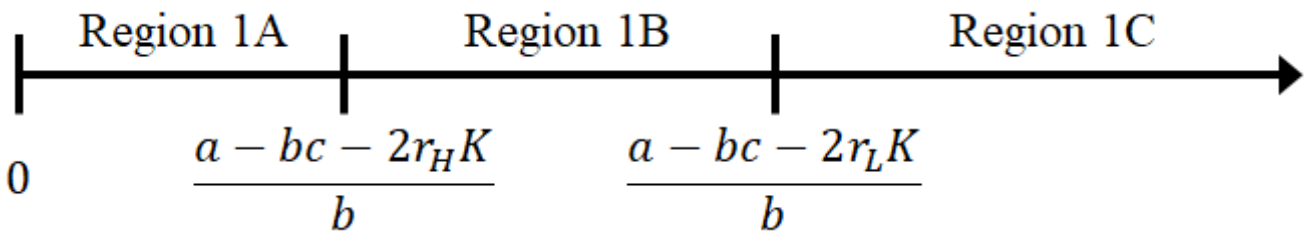
- In any separating equilibrium, the unreliable vendor offers her first-best contract.
  - $f_L^F = \frac{(a-bc)^2}{4b}$ if $r_L K \geq \frac{a-bc}{2}$
  - $f_L^F = \frac{(a-bc-r_L K) r_L K}{b}$ otherwise.
- For the reliable vendor to separate from the unreliable one, she must offer a revenue-sharing contract.
- Below we will show that there is no value of w_H^F that may satisfy all required constraints at the same time under some condition.
  - As the unreliable vendor's behavior depends on whether $r_L K \geq \frac{a-bc}{2}$, we divide the proof into two cases.

Proof of Proposition 2 (Case 1)

- The per-treatment fee w_H^F must satisfy:
  - (IR) The hospital earns a nonnegative profit.
  - (IC-L) The unreliable vendor has no incentive to mimic the reliable one.
  - (IC-H) The reliable vendor has no incentive to mimic the unreliable one.
- Let p^* be the equilibrium treatment price, the three constraints are

$$\begin{aligned} (p^* - w_H^F - c)\min\{a - bp^*, r_H K\} &\geq 0 && \text{(IR)} \\ f_L^F &\geq w_H^F \min\{a - bp^*, r_L K\} && \text{(IC-L)} \\ w_H^F \min\{a - bp^*, r_H K\} &\geq f_L^F. && \text{(IC-H)} \end{aligned}$$

- We need to examine the feasibility of w_H^F in three regions:



Proof of Proposition 2 (Region 1A)

- If $w_H^F \leq \frac{a-bc-2r_HK}{b}$, we have $p^* = \frac{a-r_HK}{b}$, $\min\{a-bp^*, r_HK\} = r_HK$, and $\min\{a-bp^*, r_LK\} = r_LK$.
- The three constraints become

$$\left(\frac{a-r_HK}{b} - w_H^F - c\right) r_HK \geq 0 \quad \text{(IR)}$$

$$\frac{(a-bc-r_LK)r_LK}{b} \geq w_H^F r_LK \quad \text{(IC-L)}$$

$$w_H^F r_HK \geq \frac{(a-bc-r_LK)r_LK}{b}. \quad \text{(IC-H)}$$

- To satisfy (IR) and (IC-H) together, w_H^F should fall in the interval $[(\frac{a-bc-r_LK}{b})\frac{r_L}{r_H}, \frac{a-bc-r_HK}{b}]$. This is impossible if the left endpoint is greater than the right one, which happens if $K > \frac{a-bc}{r_H+r_L}$.

Proof of Proposition 2 (Region 1B)

- If $\frac{a-bc-2r_HK}{b} < w_H^F \leq \frac{a-bc-2r_LK}{b}$, we have $p^* = \frac{a+bc+bw_H^F}{2b}$, $\min\{a-bp^*, r_HK\} = \frac{a-bc-bw_H^F}{2}$, and $\min\{a-bp^*, r_LK\} = r_LK$.
- The (IR) constraint becomes

$$\left(\frac{a+bc+bw_H^F}{2b} - w_H^F - c\right)\left(\frac{a-bc-bw_H^F}{2}\right) \geq 0.$$

- (IC-L) remain the same as in Case 1A, and (IC-H) becomes

$$w_H^F\left(\frac{a-bc-bw_H^F}{2}\right) \geq \frac{(a-bc-r_LK)r_LK}{b}.$$

- There is no value satisfying (IC-H) if $(\frac{a-bc}{2})^2 - 4(\frac{b}{2})(\frac{a-bc-r_Lk}{b})r_Lk < 0$. Hence, separation is impossible if $4-2\sqrt{2} < \frac{a-bc}{r_Lk} < 4+2\sqrt{2}$.

Proof of Proposition 2 (Region 1C)

- If $w_H^F \geq \frac{a-bc-2r_LK}{b}$, we have p^* and $\min\{a-bp^*, r_HK\}$ unchanged. However, $\min\{a-bp^*, r_LK\} = \frac{a-bc-bw_H^F}{2}$.
- (IR) and (IC-H) remain the same as those in Region 1B, and (IC-L) becomes

$$\frac{(a-bc-r_LK)r_LK}{b} \geq w_H^F\left(\frac{a-bc-bw_H^F}{2}\right).$$

- It turns out that (IC-L) does not matter.
- The condition derived in Region 1B based on (IC-H) still applies.

Proof of Proposition 2 (Case 1)

- Collectively, when $r_L K \leq \frac{a-bc}{2}$ is true,
  - If $K > \frac{a-bc}{r_H + r_L}$, no w_H^F satisfies all three constraints in Region 1A.
  - If $4 - 2\sqrt{2} < \frac{a-bc}{r_L k} < 4 + 2\sqrt{2}$, no w_H^F satisfies all three constraints in Regions 1B and 1C.
- If the two conditions are satisfied at the same time, there is no $w_H^F \in [0, \infty)$ that may satisfy all three constraints.
- Therefore, when $r_L K \leq \frac{a-bc}{2}$, separation is impossible if

$$2r_L K \leq a - bc < \min\left\{4 + 2\sqrt{2}, \frac{r_H}{r_L} + 1\right\} r_L K.$$

Proof of Proposition 2

- For the second case $r_L K > \frac{a-bc}{2}$, we may follow the same way to check for feasibility.
  - In all the three regions, no w_H^F satisfies all constraints.
  - Separation is impossible as long as $a - bc < 2r_L K$.
- Based on the analyses for Cases 1 and 2, we show that separation is impossible if either

$$2r_L K \leq a - bc < \min\left\{4 + 2\sqrt{2}, \frac{r_H}{r_L} + 1\right\} r_L K \quad \text{or} \quad a - bc < 2r_L K.$$

- Combining these two conclusions, we conclude that a separating equilibrium does not exist if

$$a - bc < \min\left\{4 + 2\sqrt{2}, \frac{r_H}{r_L} + 1\right\} r_L K.$$

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Comparisons

- When selling to a **non-profit** hospital, a separating equilibrium **always exists**.
  - The treatment price is exogenous.
  - Both a fixed fee and a per-treatment payment extract full surplus.
  - Reliability always affects the treatment volume.
- When selling to a **for-profit** hospital, it is possible that a separating equilibrium **does not exist**.
  - The treatment price is endogenous.
  - Only a fixed fee can extract full surplus.
  - A per-treatment payment drives up the treatment price, drives down the demand, and makes reliability less critical for the treatment volume.
  - *When the vendor is able to reveal the true information, that true information becomes less important to be revealed.*

Conclusions

- The equipment vendor has a **a lower incentive** to rent the machine to private hospitals than to public ones.
  - When it is a public hospital, a revenue-sharing contract signals the high reliability and increase the reliable vendor's expected profit.
  - When it is a private hospital, it is worthwhile to signal reliability through a revenue-sharing contract only if the **efficiency loss** is not severe.
- Regarding this research:
  - We observe different entities **acting differently** in practice.
  - There are obvious reasons. We look for **non-obvious reasons**.
  - An empirical study helps us identify potential factors.
  - A theoretical study helps us find explanations.