Information Economics, Fall 2015
Homework 2

Instructor: Ling-Chieh Kung
Department of Information Management
National Taiwan University

Note. DO NOT submit this homework. This problems are only for you to practice by yourselves. Doing these problems definitely help you understand course materials more and increase your chance for getting good grades in exams. The solution will be provided on October 16, 2014. Of course, you are more than welcome to discuss these problems with the instructor or TA.

1. Consider McGuire and Staelin (1983). While the Bertrand competition under pure integration makes the prices too low, switching to pure decentralization drives the prices up. A natural question is: Is it possible for the prices to go back to the efficient level? In this problem, we will address this issue.

   (a) Find $p^*(\theta)$, the efficient price under complete integration for $\theta \in [0, 1]$. This can be found by assuming that the two manufacturer collude to choose a price to maximize industry profit.

   (b) For $\theta \in [0, 0.9]$, draw the following three curves: (1) $p^*(\theta)$, (2) $p^{\mathrm{I}}(\theta)$, the equilibrium price under pure integration (II), and (3) $p^{\mathrm{D}}(\theta)$, the equilibrium price under pure decentralization (DD).[1] Compare $p^{\mathrm{D}}(\theta)$ and $p^*(\theta)$. When is $p^{\mathrm{D}}(\theta)$ higher and when is $p^*(\theta)$ higher?

   (c) Does adding one level of intermediary drive the price back to the efficient level? If so, under what value(s) of θ?

2. In Theorem 1 of Pasternack (1985), we know full returns with full credits will not coordinate the channel. In other words, the retailer will be induced to order an inefficient (i.e., system-suboptimal) inventory level. Is the equilibrium inventory level too low or too high? Provide both derivations and economic intuitions for your answer.

3. Consider an indirect newsvendor channel with production cost c, retail price p, and random demand $D \sim f, F$. Suppose the manufacturer offers the retailer the following *two-part tariff* (w, t): The manufacturer charges the retailer a per unit wholesale price w plus a lump sum fixed payment t.[2] The retailer then chooses an order quantity q by paying $wq + t$ to the manufacturer.

   (a) How is a wholesale contract with wholesale price w a special case of the two-part tariff (w, t)?

   (b) Formulate the retailer's expected profit if he orders q units.

   (c) Suppose the retailer will be willing to accept the contract if and only if she can earn no less than that with a wholesale price contract. Formulate the manufacturer's problem of maximizing her own expected profit.

   (d) Can this two-part tariff achieves channel coordination? If yes, can it achieve win-win?

4. Consider an indirect newsvendor channel with production cost c, retail price p, and random demand $D \sim f, F$. Suppose the manufacturer offers the retailer the following *two-part tariff* (q, t): The manufacturer delivers q units of the product to the retailer in exchange of a fixed payment t from the retailer. Note that the payment t is a *lump sum* payment, not a per unit payment. The retailer can only choose to accept or reject the contract.[3] If she accepts, she must pay t and get q units. Otherwise, both firms will earn no profit.

   (a) How is a wholesale contract with wholesale price w a special case of a two-part tariff (q, t)?

   (b) Formulate the retailer's expected profit if he accepts the contract.

[1] You should see that $p^*(\theta) > p^{\mathrm{I}}(\theta)$ and $p^D(\theta) > p^I(\theta)$, which verify our intuitions that horizontal decentralization (from colluding to competition) drives down the price and vertical decentralization (from II to DD) drives up the price.

[2] The fixed payment t is generally called a *transfer* in the economics literature.

[3] Such an offer is called a *take-it-or-leave-it* contract.

(c) Suppose the retailer will be willing to accept the contract if and only if she can earn no less than that with a wholesale price contract. Formulate the manufacturer's problem of maximizing her own expected profit.

(d) Can this two-part tariff achieves channel coordination? If yes, can it achieve win-win?

5. Consider the following dynamic game between the owner of a retail store and a worker in the store. We will call the owner "the retailer". Suppose the retailer only sells one product, whose production cost is normalized to 0. In each day, the retailer first chooses a retail price p and then the worker chooses a service level a. Given p and a, the demand of this product is $1 - p + a$. In other words, better services attract more consumers. However, better services do not come with no cost. Suppose the cost for offering service level a is $\frac{1}{2}a^2$. Both the retailer and the worker want to maximize their own profit. The retailer's profit is the sales revenue minus the worker's salary. The worker's profit is his salary minus his cost of services.

   (a) Suppose the retailer offers the worker a fixed payment $t \geq 0$. Show that the worker will provide the minimum service, i.e., the equilibrium service level is 0. Then find the equilibrium price and the two players' equilibrium profits.

   (b) Suppose the retailer shares the sales revenue with the worker by setting a commission rate $v \in [0, 1]$. If the total sales revenue is π, the retailer gets $(1 - v)\pi$ and the worker gets $v\pi$. Given the announced v and p, what is the equilibrium service level?

   (c) Continue from Part (b), what are the equilibrium price and commission rate?[4] What are the equilibrium profits of the two players?

   (d) Compare your answers in Part (a) and (c). Does the contract with a commission rate increase both players' profits? Does it decrease both players' profits? Does it make one player better off and the other worse off?

   (e) Now consider the first best case, in which the retailer "integrates" the worker.[5] What are the efficient price and service level? What is the system profit?

   (f) Compare your answers in Part (c) and (e). Is the contract with a commission rate efficient, i.e., making the two players together earn the system profit? Is the equilibrium price efficient? Is the equilibrium service level efficient? If no for either question, is it too high or too low? Given economic arguments (i.e., intuitions) to explain why.

References

McGuire, T. W., R. Staelin. 1983. An industry equilibrium analysis of downstream vertical integration. *Marketing Science* **2**(1) 115–130.

Pasternack, B. 1985. Optimal pricing and return policies for perishable commodities. *Marketing Science* **4**(2) 166–176.

[4] To solve this two-dimensional optimization problem, you need to apply the FOC twice, once for v and once for p.

[5] You may imagine that the worker is the son of the retailer and they together determine the retail price and service level to maximize system profit.