Information Economics

Channel Coordination with Returns

Ling-Chieh Kung

Department of Information Management
National Taiwan University

Road map

- **Introduction**.
- Return contracts.
- Model and analysis.
- Insights and conclusions.

When centralization is impossible

- We hope people all cooperate to maximize social welfare and then fairly allocate payoffs.
- Complete **centralization**, or **integration**, is the best.
- However, it may be impossible.
  - Each person has her/his **self interest**.
- Facing a **decentralized** system, we will not try to integrate it.
  - We will not assume (or try to make) that people act for the society.
  - We will assume that people are all **selfish**.
  - We seek for **mechanisms** to improve the efficiency.
  - This is **mechanism design**.

Issues under decentralization

- What issues arise in a decentralized system?
- The **incentive** issue:
  - Workers need incentives to work hard.
  - Students need incentives to keep labs clean.
  - Manufacturers need incentives to improve product quality.
  - Consumers need incentives to pay for a product.
- The **information** issue:
  - Efforts of workers and students are hidden.
  - Product quality and willingness-to-use are hidden.
- Information issues **amplify** or even **create** incentive issues.

Incentive alignment

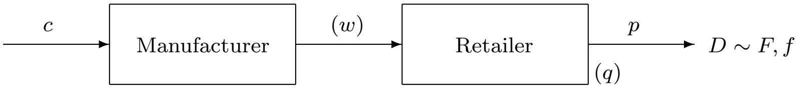
- One typical goal is to **align** the incentives of different players.
- As an example, an employer wants her workers to work as hard as possible, but a worker always prefers vacations to works.
  - There is **incentive misalignment** between the employer and employee.
  - To better align their incentives, the employer may put what the employee cares into the employee's utility function.
  - This is why we see sales bonuses and commissions!

Double marginalization

- In a supply chain or distribution channel, incentive misalignment may cause **double marginalization**.
- Consider the pricing in a supply chain problem:
  - The unit cost is c.
  - The manufacturer charges $w^* > c$ with one layer of "marginalization".
  - The retailer charges $r^* > w^*$ with another layer of marginalization.
  - The equilibrium retail price r^* is **too high**. Both firms are hurt.
- The system is **inefficient** because the equilibrium decisions (retail price) is **system-suboptimal** (in this case, too high).

Inventory and newsvendor

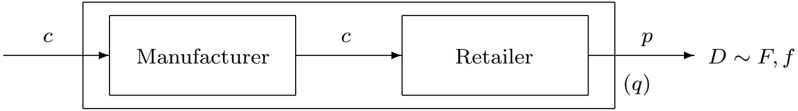
- Consumer demands are not always certain.
- Let's assume that the retailer is a price taker and makes **inventory** decisions for **perishable** products.



- Decisions:
  - The manufacturer chooses the **wholesale price** w.
  - The retailer, facing uncertain demand $D \sim F, f$ and fixed retail price p, chooses the **order quantity** (inventory level) q.
  - Assumption: $D \geq 0$ and is continuous: $F' = f$.
- They try to maximize:
  - The retailer: $\pi_{\text{R}}(q) = p\mathbb{E}[\min\{D, q\}] - wq$.
  - The manufacturer: $\pi_{\text{M}}(w) = (w - c)q^*$, where $q^* \in \text{argmax}_q\{\pi_{\text{R}}(q)\}$.

Efficient inventory level

- Suppose the two firms integrate:



- They choose q to maximize $\pi_{\text{C}}(q) = p\mathbb{E}[\min\{D, q\}] - cq$.

Proposition 1

The efficient inventory level q^{FB} satisfies $F(q^{FB}) = 1 - \frac{c}{p}$.

Proof. Because $\pi_{\text{C}}(q) = r\{\int_0^q xf(x)dx + \int_q^\infty qf(x)dx\} - cq$, we have $\pi'_{\text{C}}(q) = r[1 - F(q)] - c$ and $\pi''_{\text{C}}(q) = -rf(q) \leq 0$. Therefore, $\pi_{\text{C}}(q)$ is concave and $\pi'_{\text{C}}(q^{\text{FB}}) = 0$ is the given condition. □

Retailer-optimal inventory level

- The retailer maximizes $\pi_{\mathrm{R}}(q) = p\mathbb{E}[\min\{D, q\}] - wq$.
- Let $q^* \in \operatorname{argmax}_{q \geq 0} \pi_{\mathrm{R}}(q)$ be the retailer-optimal inventory level.

Proposition 2

We have $q^* < q^{FB}$ if F is strictly increasing.

Proof. Similar to the derivation for q^{FB}, we have $F(q^*) = 1 - \frac{w}{p}$ given any wholesale price w. Note that $F(q^*) = 1 - \frac{w}{p} < 1 - \frac{c}{p} = F(q^{\mathrm{FB}})$ if $w > c$, which is true in any equilibrium. Therefore, once F is strictly increasing, we have $q^* < q^{\mathrm{FB}}$. □

- **Decentralization** again introduces **inefficiency**.
  - Similar to double marginalization.

What should we do?

- How to reduce inefficiency?
- Complete integration is the best but impractical.
- We may make these player **interacts** in a **different** way.
  - We may change the "game rules".
  - We may design different mechanisms.
  - We want to **induce satisfactory behaviors**.
- In this lecture, we will introduce a seminal example of redesigning a mechanism to enhance efficiency.
  - We change the **contract format** between two supply chain members.
  - This belongs to the fields of **supply chain coordination** or **supply chain contracting**.

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How to help the indirect newsvendor?

- What happened in the indirect newsvendor problem?
  - The inventory level (order/production/supply quantity) is **too low**.
  - The inventory level is optimal for the retailer but too low for the system.
- Why the retailer orders an inefficiently low quantity?
- Demand is uncertain:
  - The retailer takes all the **risks** while the manufacturer is **risk-free**.
  - When the unit cost increases (from c to w), overstocking becomes more harmful. The retailer thus lower the inventory level.
- How to induce the retailer to order more?
  - Reducing the wholesale price? No way!
  - A practical way is for the manufacturer to **share the risk**.
  - Pasternack (1985) studies **return** (buy-back) contracts.[1]

[1] Pasternack, B. 1985. Optimal pricing and return policies for perishable commodities. *Marketing Science* **4**(2) 166–176.

Why return contracts?

- A **return** (buy-back) contract is a **risk-sharing** mechanism.
- When the products are not all sold, the retailer is allowed to return (all or some) unsold products to get credits.
- Contractual terms:
  - w is the wholesale price.
  - r is the buy-back price (return credit).
  - R is the percentage of products that can be returned.
- Several alternatives:
  - Full return with full credit: $R = 1$ and $r = w$.
  - Full return with partial credit: $R = 1$ and $r < w$.
  - Partial return with full credit: $R < 1$ and $r = w$.
  - Partial return with partial credit: $R < 1$ and $r < w$.
- Before we jump into the analytical model, let's get the idea with a numerical example.

A numerical example

- Consider a distribution channel in which a manufacturer (she) sells a product to a retailer (he), who then sells to end consumers.
- Suppose that:
  - The unit production cost is \$10.
  - The unit retail price is \$50.
  - The random demand follows a uniform distribution between 0 and 100.

Benchmark: integration

- As a benchmark, let's first find the **efficient inventory level**, which will be implemented when the two firms are integrated.
- Let Q^*_{T} be the efficient inventory level that maximizes the expected system profit, we have

$$\frac{Q^*_{\mathrm{T}}}{100} = 1 - \frac{10}{50} \quad \Rightarrow \quad Q^*_{\mathrm{T}} = 80.$$

- The expected system profit, as a function of Q, is

$$\begin{aligned}\pi_{\mathrm{T}}(Q) &= 50\left\{\int_0^Q x\left(\frac{1}{100}\right)dx + \int_Q^{100} Q\left(\frac{1}{100}\right)dx\right\} - 10Q \\ &= -\frac{1}{4}Q^2 + 40Q.\end{aligned}$$

- The optimal system profit is $\pi^*_{\mathrm{T}} = \pi_{\mathrm{T}}(Q^*_{\mathrm{T}}) = \$1600$.

Wholesale contract

- Under the wholesale contract, we have the indirect newsvendor problem.
- We know that in equilibrium, the manufacturer sets the wholesale price $w^* = \frac{50+10}{2} = 30$ and the retailers orders $Q^*_{\mathrm{R}} = 40$.
- The retailer's expected profit, as a function of Q, is

$$\begin{aligned}\pi_{\mathrm{R}}(Q) &= 50\left\{\int_0^Q x\left(\frac{1}{100}\right)dx + \int_Q^{100} Q\left(\frac{1}{100}\right)dx\right\} - 30Q \\ &= -\frac{1}{4}Q^2 + 20Q.\end{aligned}$$

- The retailer's expected profit is $\pi^*_{\mathrm{R}} = \pi_{\mathrm{R}}(Q^*_{\mathrm{R}}) = \$400$.
- The manufacturer's expected profit is $\pi^*_{\mathrm{M}} = 40 \times (30 - 10) = \$800$.
- The expected system profit is $\pi^*_{\mathrm{R}} + \pi^*_{\mathrm{M}} = \$1200 < \pi^*_{\mathrm{T}} = \$1600$.

Return contract 1

- Consider the following return contract:
  - The wholesale price $w = 30$.
  - The return credit $r = 5$.
  - The percentage of allowed return $R = 1$.
- The retailer's expected profit, as a function of Q, is

$$\pi_{\mathrm{R}}^{(1)}(Q) = 50\left\{ \int_0^Q \frac{x}{100}dx + \int_Q^{100} \frac{Q}{100}dx \right\} + 5\int_0^Q \frac{Q-x}{100}dx - 30Q$$
$$= -\frac{1}{4}Q^2 + \frac{1}{40}Q^2 + 20Q \quad \Rightarrow \quad Q_{\mathrm{R}}^{(1)} = \frac{400}{9} \approx 44.44.$$

- The retailer's expected profit is $\pi_{\mathrm{R}}^{(1)} = \pi_{\mathrm{R}}(Q_{\mathrm{R}}^{(1)}) \approx \$444.44 > \pi_R^*$.
- The manufacturer's expected profit is $\pi_{\mathrm{M}}^{(1)} = (\frac{400}{9})(30 - 10) - \frac{4000}{81} \approx 888.89 - 49.38 = \$839.51 > \pi_{\mathrm{M}}^*$.
- The expected system profit is $\pi_{\mathrm{R}}^{(1)} + \pi_{\mathrm{M}}^{(1)} = \$1283.95 < \pi_{\mathrm{T}}^* = \$1600$.

Return contract 2

- Consider a more generous return contract:
  - The wholesale price $w = 30$.
  - The return credit $r = 10$.
  - The percentage of allowed return $R = 1$.
- The retailer's expected profit, as a function of Q, is

$$\pi_{\mathrm{R}}^{(2)}(Q) = 50\left\{\int_0^Q \frac{x}{100}dx + \int_Q^{100} \frac{Q}{100}dx\right\} + 10\int_0^Q \frac{Q-x}{100}dx - 30Q$$
$$= -\frac{1}{4}Q^2 + \frac{1}{20}Q^2 + 20Q \quad \Rightarrow \quad Q_{\mathrm{R}}^{(2)} = 50.$$

- The retailer's expected profit is $\pi_{\mathrm{R}}^{(2)} = \pi_{\mathrm{R}}(Q_{\mathrm{R}}^{(2)}) = \$500 > \pi_R^{(1)}$.
- The manufacturer's expected profit is $\pi_{\mathrm{M}}^{(2)} = 50 \times (30 - 10) - 125 \approx 1000 - 125 = \$875 > \pi_{\mathrm{M}}^{(1)}$.
- The expected system profit is $\pi_{\mathrm{R}}^{(2)} + \pi_{\mathrm{M}}^{(2)} = \$1375 < \pi_{\mathrm{T}}^* = \$1600$.

Comparison

- The **performance** of these contracts:

(w, r, R)	Q	π_{R}	π_{M}	$\pi_{\text{R}} + \pi_{\text{M}}$
$(30, 0, 1)$	40	400	800	1200
$(30, 5, 1)$	44.44	444.44	839.51	1283.95
$(30, 10, 1)$	50	500	875	1375
Efficient	80	–	–	1600

  - Will Q keep increasing when r increases?
  - Will π_{R} and π_{M} keep increasing when r increases?
  - Will $Q = Q^*_{\text{T}} = 80$ for some r? Will $\pi_{\text{R}} + \pi_{\text{M}} = \pi^*_{\text{T}} = 1600$ for some r?
- There are so many questions!
  - What if $w \neq 30$? What if $R < 1$?
  - What if the demand is not uniform?
- When may we achieve **channel coordination**, i.e., $Q = Q^*_{\text{T}} = 80$?
- We need a general analytical model to really deliver insights.

Road map

- Introduction.
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- **Model and analysis**.
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Model

- We consider a manufacturer-retailer relationship in an indirect channel.
- The product is perishable and the single-period demand is random.
- Production is under MTO and the retailer is a newsvendor.
- We use the following notations:

Symbol	Meaning
c	Unit production cost
w	Unit wholesale price
r	Unit return credit
R	Percentage of allowed return
Q	Order quantity
F	Distribution function of demand
f	Density function of demand

- Assumptions:
  - $c < w < p$; $r \leq w$; f is continuous; $f(x) = 0$ for all $x < 0$.

Utility functions

- Under the return contract (w, r, R), the retailer's expected profit is

$$\pi_{\mathrm{R}}(Q) = -Qw + \int_0^{(1-R)Q} (xp + RQr) f(x) dx + \int_{(1-R)Q}^{Q} \Big[xp + (Q - x)r \Big] f(x) dx + \int_Q^{\infty} Qpf(x) dx.$$

- The manufacturer's expected profit is

$$\pi_{\mathrm{M}}(Q) = Q(w - c) - \int_0^{(1-R)Q} RQrf(x) dx - \int_{(1-R)Q}^{Q} (Q - x) rf(x) dx.$$

- The expected system profit is

$$\pi_{\mathrm{T}}(Q) = -cQ + \int_0^{Q} xpf(x) dx + \int_Q^{\infty} Qpf(x) dx.$$

Timing

- First a return contract is signed by the manufacturer and retailer.
- Then the retailer places an order.
- The manufacturer produces and ships products to the retailer.
- The sales season starts, the demand is realized, and the allowed unsold products (if any) are returned to the manufacturer.

System-optimal (efficient) inventory level

- The expected system profit is

$$\pi_{\mathrm{T}}(Q) = -cQ + \int_0^Q xpf(x)dx + \int_Q^\infty Qpf(x)dx.$$

- The system optimal inventory level Q_T^* satisfies the equation

$$F(Q_{\mathrm{T}}^*) = 1 - \frac{c}{p}.$$

- We hope that there is a return contract (w, r, R) that makes the retailer order Q_{T}^*.

Retailer's ordering strategy

- Under the return contract, the retailer's expected profit is

$$\pi_{\mathrm{R}}(Q) = -Qw + \int_0^{(1-R)Q} (xp + RQr) f(x)dx + \int_{(1-R)Q}^{Q} \Big[xp + (Q-x)r\Big] f(x)dx + \int_Q^{\infty} Qpf(x)dx.$$

- Let's differentiate it... How?!?!?!
- We need the Leibniz integral rule: Suppose $f(x,y)$ is a function such that $\frac{\partial}{\partial y} f(x,y)$ exists and is continuous, then we have

$$\frac{d}{dy}\int_{a(y)}^{b(y)} f(x,y)dx = f(b(y),y)b'(y) - f(a(y),y)a'(y) + \int_{a(y)}^{b(y)} \frac{\partial}{\partial y} f(x,y)dx$$

Retailer's ordering strategy

- Let's apply the Leibniz integral rule

$$\frac{d}{dy}\int_{a(y)}^{b(y)} f(x,y)dx = f(b(y),y)b'(y) - f(a(y),y)a'(y) + \int_{a(y)}^{b(y)} \frac{\partial}{\partial y} f(x,y)dx$$

to the retailer's expected profit function $\pi_{\mathrm{R}}(Q)$:

Inside $\pi_{\mathrm{R}}(Q)$	Inside $\pi'_{\mathrm{R}}(Q)$
$-Qw$	$-w$
$\int_0^{(1-R)Q} (xp + RQr) f(x)dx$	$(1-R)\Big[(1-R)Qp + RQr\Big] f\Big((1-R)Q\Big)$ $+ \int_0^{(1-R)Q} Rrf(x)dx$
$\int_{(1-R)Q}^{Q} \Big[xp + (Q-x)r\Big] f(x)dx$	$Qpf(Q)$ $-(1-R)\Big[(1-R)Qp - RQr\Big] f\Big((1-R)Q\Big)$ $+ \int_{(1-R)Q}^{Q} rf(x)dx$
$\int_Q^{\infty} Qpf(x)dx$	$-Qpf(Q) + \int_Q^{\infty} pf(x)dx$

Retailer's ordering strategy

- We then have

$$\begin{aligned}\pi'_{\mathrm{R}}(Q) &= -w + \int_0^{(1-R)Q} Rrf(x)dx + \int_{(1-R)Q}^{Q} rf(x)dx + \int_Q^{\infty} pf(x)dx \\ &= w + RrF\Big((1-R)Q\Big) + r\Big[F(Q) - F\Big((1-R)Q\Big)\Big] + p\Big[1 - F(Q)\Big] \\ &= -w + p - (p-r)F(Q) - (1-R)rF\Big((1-R)Q\Big).\end{aligned}$$

- Given (w, r, R), the retailer may numerically search for Q^*_{R} that satisfies $\pi'_{\mathrm{R}}(Q^*_{\mathrm{R}}) = 0$. This is the retailer's ordering strategy.
  - Why $\pi'_{\mathrm{R}}(Q) = 0$ always has a unique root?

Inducing the system-optimal inventory level

- The system-optimal inventory level Q^*_T satisfies

$$F(Q^*_{\mathrm{T}}) = 1 - \frac{c}{p} = \frac{p-c}{p}.$$

- To induce the retailer to order Q^*_{T}, we must make Q^*_{T} optimal for the retailer. Therefore, we need $\pi'_{\mathrm{R}}(Q^*_{\mathrm{T}}) = 0$, i.e.,

$$\begin{aligned}\pi'_{\mathrm{R}}(Q^*_{\mathrm{T}}) &= -w + p - (p-r)F(Q^*_{\mathrm{T}}) - (1-R)rF\Big((1-R)Q^*_{\mathrm{T}}\Big)\\ &= -w + p - \frac{(p-c)(p-r)}{p} - (1-R)rF\Big((1-R)Q^*_{\mathrm{T}}\Big) = 0.\end{aligned}$$

- To achieve coordination, we need to choose (w, r, R) to make the above equation hold, where Q^*_{T} is uniquely determined by $F(Q^*_{\mathrm{T}}) = \frac{p-c}{p}$.
- Is it possible?

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Extreme case 1: full return with full credit

$$\pi'_{\mathrm{R}}(Q^*_{\mathrm{T}}) = w - p + \frac{(p-c)(p-r)}{p} + (1-R)rF\Big((1-R)Q^*_{\mathrm{T}}\Big).$$

- Let's consider the most generous return contract.

Proposition 3

If $r = w$ and $R = 1$, $\pi'_{\mathrm{R}}(Q^*_{\mathrm{T}}) = 0$ if and only if $c = 0$.

Proof. If $r = w$ and $R = 1$, $\pi'_{\mathrm{T}}(Q^*_{\mathrm{T}}) = 0$ becomes

$$w - p + \frac{(p-c)(p-w)}{p} = (p-w)\left(\frac{p-c}{p} - 1\right) = 0.$$

As $p > w$, we need $\frac{p-c}{p} = 1$, i.e., $c = 0$. □

- Allowing full returns with full credits is generally system suboptimal.

Extreme case 2: no return

$$\pi'_{\mathrm{R}}(Q^*_{\mathrm{T}}) = w - p + \frac{(p-c)(p-r)}{p} + (1-R)rF\Big((1-R)Q^*_{\mathrm{T}}\Big).$$

- Let's consider the least generous return contract.

Proposition 4

If $r = 0$ or $R = 0$, $\pi'_{\mathrm{R}}(Q^*_{\mathrm{T}}) = 0$ is impossible.

Proof. If $r = 0$, $\pi'_{\mathrm{R}}(Q^*_{\mathrm{T}}) = 0$ becomes $w - c = 0$, which cannot be true. If $R = 0$, it becomes

$$w - p + \frac{(p-c)(p-r)}{p} + rF(Q^*_{\mathrm{T}}) = w - c = 0,$$

which is also impossible. □

- Allowing no return is system suboptimal.

Full returns with partial credits

$$\pi'_{\mathrm{R}}(Q^*_{\mathrm{T}}) = w - p + \frac{(p-c)(p-r)}{p} + (1-R)rF\Big((1-R)Q^*_{\mathrm{T}}\Big).$$

- Let's consider full returns with partial credits.

Proposition 5

- *If* $R = 1$, $\pi'_{\mathrm{R}}(Q^*_{\mathrm{T}}) = 0$ *if and only if* $w = p - \frac{(p-c)(p-r)}{p}$.
- *For any* p *and* c, *a pair of* r *and* w *such that* $0 < r < w$ *can always be found to satisfy the above equation.*

Proof. When $R = 1$, the first part is immediate. According to the equation, we need $r = \frac{p(w-c)}{p-c}$. Then $w < p$ implies $\frac{p(w-c)}{p-c} < w$ and $c < w$ implies $\frac{p(w-c)}{p-c} > 0$. □

- Allowing full returns with partial credits can be system optimal!
- In this case, we say the return contract **coordinates** the system.

Profit splitting

- Under a full return contract, channel coordination requires

$$w = p - \frac{(p-c)(p-r)}{p} = c + \left(\frac{p-c}{p}\right)r.$$

  - The expected system profit is maximized. The "pie" is maximized.
  - Do both players benefit from the enlarged pie?
  - To ensure win-win, we hope the pie can be split **arbitrarily**.
- In one limiting case (though not possible), when $w = c$, we need $r = 0$. In this case, $\pi^*_{\mathrm{M}} = 0$ and $\pi^*_{\mathrm{R}} = \pi^*_{\mathrm{T}}$.
- In another limiting case, when $w = p$, we need $r = p$. In this case, $\pi^*_{\mathrm{M}} = \pi^*_{\mathrm{T}}$ and $\pi^*_{\mathrm{R}} = 0$.
- How about the intermediate cases?

Profit splitting

- Let's visualize the set of coordinating full return contracts:

- As $\pi_{\mathrm{T}}(\cdot)$ is continuous in w and r, π^*_{M} must **gradually** go up from 0 to π^*_{T} as w goes from c to p.
  - π^*_R must gradually do down as w goes from p to c.
  - **Arbitrary profit splitting** can be done!

Coordination and win-win

- We know that return contracts can be **coordinating**.
  - We can make the inventory level efficient.
  - We can make the channel efficient.
- Now we know they can also be **win-win**.
  - We can split the pie in any way we want.
  - We can always make both players happy.
- The two players will **agree** to adopt a coordinating return contract.
- Consumers also benefit from channel coordination. Why?
- Some remarks:
  - Not all coordinating contracts are win-win.
  - In practice, the manufacturer may pay the retailer without asking for the physical goods. Why?

More in the paper

- We only introduced the main idea of the paper.
- There are still a lot untouched:
  - Salvage values and shortage costs.
  - Monotonicity of the manufacturer's and retailer's expected profit.
  - Environments with multiple retailers.
- Read the paper by yourselves.
- Studying contracts that coordinate a supply chain or distribution channel is the theme of the subject **supply chain coordination**.
  - It was a hot topic in 1980's and 1990's.
  - Not so hot now.
- Other contracts to coordinate a channel or a supply chains:
  - Two-part tariffs.
  - Quantity flexible contracts.
  - Revenue-sharing contracts.
  - Sales rebate contracts.