Operations Research, Spring 2013
Homework 05

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1. (10 points) Write down the number of your courses that have a midterm exam between April 15th to 19th.

2. (Modified from Problem 4.7.3; 10 points) Consider the following LP

$$\begin{array}{rrl} z^* = & \max & x_1 + x_2 \\ & \text{s.t.} & x_1 + x_2 + x_3 \leq 1 \\ & & x_1 + 2x_3 \leq 1 \\ & & x_i \geq 0 \quad \forall\, i = 1, ..., 3. \end{array}$$

   Use the simplex method to determine whether the LP is infeasible, unbounded, having a unique optimal solution, or having multiple optimal solutions. In the latter two cases, find the optimal solution if there is only one or find two if there are multiple.

   Note. The suggested way of checking whether there are multiple optimal solutions is to investigate the final tableau. You should not start from the initial tableau twice and try different entering variables.

3. (Modified from Problem 4.7.8; 15 points, 5 points each) Suppose when we run the simplex method for a given linear program with a maximization objective function, a tableau we get is

$$\begin{array}{cccc|c} 0 & 0 & 0 & 2 & 2 \\ \hline 1 & 0 & -1 & 1 & x_1 = 6 \\ 0 & 1 & -2 & 3 & x_2 = 3 \end{array}$$

   Answer the following questions with brief explanations.

   (a) Is this linear program unbounded? Why?
   (b) Are there multiple optimal solutions? Why?
   (c) How many optimal basic feasible solutions do we have? Why?

4. (Modified from Problem 4.8.1; 10 points) Consider the following LP

$$\begin{array}{rrl} z^* = & \max & 2x_2 \\ & \text{s.t.} & x_1 - x_2 \leq 4 \\ & & -x_1 + x_2 \leq 1 \\ & & x_i \geq 0 \quad \forall\, i = 1, 2. \end{array}$$

   Use the simplex method to determine whether the LP is infeasible, unbounded, having a unique optimal solution, or having multiple optimal solutions. In the latter two cases, find the optimal solution if there is only one or find two if there are multiple.

5. (Modified from Problem 4.12.4; 10 points) Consider the following linear program

$$\begin{array}{rrl} z^* = & \min & 3x_2 \\ & \text{s.t.} & x_1 + 2x_2 \geq 6 \\ & & 2x_1 + 3x_2 = 4 \\ & & x_i \geq 0 \quad \forall\, i = 1, 2. \end{array}$$

   Use the simplex method to determine whether the LP is infeasible, unbounded, having a unique optimal solution, or having multiple optimal solutions. In the latter two cases, find the optimal solution if there is only one or find two if there are multiple.

6. (Modified from Problem 9.2.1; 15 points) Coach Night is trying to choose the starting lineup for the basketball team. The team consists of seven players who have been rated (on a scale of 1 for poor to 3 for excellent) according to their ball-handling, shooting, rebounding, and defensive abilities. The positions that each player is allowed to play and the player's abilities are listed in the table below. For example, player 3 can play either guard or forward.

Player	Position	Ball handling	Shooting	Rebounding	Defense
1	G	3	3	1	3
2	C	2	1	3	2
3	G or F	2	3	2	2
4	F or C	1	3	3	1
5	G or F	3	3	3	3
6	F or C	3	1	2	3
7	G or F	3	2	2	1

The five-player starting lineup must satisfy the following restrictions:

- At least 2 members must be able to play guard, at least 2 members must be able to play forward, and at least 1 member must be able to play center.
- The average rebounding level of the starting lineup must be at least 2.
- If player 3 starts, then player 6 cannot start.
- If player 1 starts, then players 4 and 5 must both start.
- Either player 2 or player 3 must start.

Given these constraints, Coach Night wants to maximize the total defensive ability of the starting team. Formulate an IP that will help him choose his starting team. You do not need to worry about whether the IP is feasible.

7. (Modified from Problem 9.2.2; 15 points) Because of excessive pollution on the Momiss River, the state of Momiss is going to build pollution control stations. Three sites (1, 2, and 3) are under consideration. Momiss is interested in controlling the pollution levels of two pollutants (1 and 2). The state legislature requires that at least 80,000 tons of pollutant 1 and at least 50,000 tons of pollutant 2 be removed from the river. The relevant data for this problem are shown in the table below.

Site	Construction cost (\$)	Treating cost (\$ / ton)	Pollutant 1 (ton)	Pollutant 2 (ton)
1	100,000	20	0.40	0.30
2	60,000	30	0.25	0.20
3	40,000	40	0.20	0.25

The first two columns record the cost of building a station and the unit cost of treating one ton of water. The last two columns record the amount of pollutants 1 and 2 that can removed by treating one ton of water. Formulate an IP to minimize the cost of meeting the state legislature's goals.

8. (Problem 9.2.10; 5 points) How could you ensure that at least one of $x + y \leq 3$ and $2x + 5y \leq 12$ is satisfied?

9. (Problem 9.2.11; 10 points) If x and y are integers, how could you ensure that whenever $x \leq 2$ is satisfied, then $y \leq 3$ is satisfied?

   Hint. The information that x is an integer is critical.