Operations Research, Spring 2013
Homework 07

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This homework is designed only for you to practice for the midterm exam. Please do not submit anything for this homework.

1. (Modified from Problem 6.5.1) Find the dual for the following LP:

$$\begin{array}{rrcrcl} \max & 2x_1 & + & x_2 & & \\ \text{s.t.} & -x_1 & + & x_2 & \leq & 1 \\ & x_1 & + & x_2 & \leq & 3 \\ & x_1 & - & 2x_2 & \leq & 4 \\ & \multicolumn{5}{l}{x_1 \geq 0,\ x_2 \geq 0.} \end{array}$$

2. (Modified from Problem 6.5.3) Find the dual for the following LP:

$$\begin{array}{rrcrcrcl} \max & 4x_1 & - & x_2 & + & 2x_3 & & \\ \text{s.t.} & x_1 & + & x_2 & & & \leq & 5 \\ & 2x_1 & + & x_2 & & & \leq & 7 \\ & & & 2x_2 & + & x_3 & \geq & 6 \\ & x_1 & & & + & x_3 & = & 4 \\ & \multicolumn{7}{l}{x_1 \geq 0,\ x_2 \text{ urs},\ x_3 \text{ urs}.} \end{array}$$

3. Consider a primal LP

$$\begin{array}{rl} \max & 3x_1 + 7x_2 + 5x_3 \\ \text{s.t.} & x_1 + x_2 + x_3 \leq 50 \\ & 2x_1 + 3x_2 + x_3 \leq 100 \\ & x_i \geq 0 \quad \forall\ i = 1, ..., 3. \end{array}$$

   (a) Use the simplex method to find an optimal tableau.
   (b) What is the primal optimal solution associated with this optimal tableau? Is it unique?
   (c) Without formulating or solving the dual program, find the shadow prices for both constraints.
   (d) Formulate the dual program, solve it graphically, and show that the values of dual variables in the dual optimal solution are indeed the primal shadow prices.

4. Consider a primal LP

$$\begin{array}{rl} \max & 2x_2 \\ \text{s.t.} & x_1 - x_2 \leq 4 \\ & -x_1 + x_2 \leq 1 \\ & x_i \geq 0 \quad \forall\ i = 1, 2. \end{array}$$

   Without solving the dual LP, show that its dual LP is infeasible.

5. (Modified from Problem 6.10.1) Glassco manufactures glasses: wine, beer, champagne, and whiskey. Each type of glass requires time in the molding shop, time in the packaging shop, and a certain amount of glass. The resources required to make each type of glass are given below.

	Wine	Beer	Champagne	Whiskey
Molding time (minutes)	4	9	7	10
Packaging time (minutes)	1	1	3	40
Glass (oz)	3	4	2	1
Selling price	\$6	\$10	\$9	\$20

Currently, 600 minutes of molding time, 400 minutes of packaging time, and 500 oz of glass are available. Assuming that Glassco wants to maximize revenue by solving the following LP

$$\begin{aligned} \max \quad & 6x_1 + 10x_2 + 9x_3 + 20x_4 \\ \text{s.t.} \quad & 4x_1 + 9x_2 + 7x_3 + 10x_4 \leq 600 \quad \text{(Molding constraint)} \\ & x_1 + x_2 + 3x_3 + 40x_4 \leq 400 \quad \text{(Packaging constraint)} \\ & 3x_1 + 4x_2 + 2x_3 + x_4 \leq 500 \quad \text{(Glass constraint)} \\ & x_i \geq 0 \quad \forall\, i = 1, ..., 4, \end{aligned}$$

where x_1, x_2, x_3, and x_4 be the production quantities of wine, beer, champagne, and whiskey, respectively. The optimal solution is $(x_1^*, x_2^*, x_3^*, x_4^*) = (\frac{400}{3}, 0, 0, \frac{20}{3})$ and the corresponding objective value is $z^* = \frac{2800}{3}$.

(a) Let y_1, y_2, and y_3 be the dual variables where y_i corresponds to primal constraint i, $i = 1, 2, 3$. Find the dual LP for the Glassco problem.

(b) Without solving the dual LP, determine the objective value associated with any dual optimal solution.

(c) Without solving the dual LP, use the given primal optimal primal solution to show that $y_3 = 0$ at any dual optimal solution.

(d) Without solving the dual LP, use the given primal optimal primal solution to show that the first dual constraint is binding at any dual optimal solution. Which other dual constraints, if any, are binding at any dual optimal solution?