Operations Research, Spring 2013
Homework 11

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1. (15 points; 5 points each) Consider the game "matching pennies"

	H	T
H	$1, -1$	$-1, 1$
T	$-1, 1$	$1, -1$

we discussed in class. For the game shown above, we know the unique mixed-strategy Nash equilibrium is for both players to choose each action with probability $\frac{1}{2}$.

   (a) Show that it is a zero-sum game.

   (b) Formulate player 1's problem of choosing a mixed strategy as a linear program.

   (c) Use the graphical approach or simplex method to solve the linear program you find in Part (b). Show that the resulting mixed strategy is indeed to choose each action with probability $\frac{1}{2}$.

2. (15 points) Suppose the game "matching pennies" is modified to

	H	T
H	$2, -1$	$-1, 1$
T	$-1, 1$	$3, -1$

   (a) (5 points) Show that there is still no pure-strategy Nash equilibrium.

   (b) (10 points) Find all the mixed-strategy Nash equilibria.

3. (Modified from Problem 1.13 in Gibbons (1992); 20 points) Each of two firms has one job opening. Suppose that the firms offer different wages: firm 1 offers \$100 per hour and firm 2 offers \$150 per hour. There are two persons, each of whom can apply to only one firm. They simultaneously decide whether to apply to firm 1 or to firm 2. If only one person applies to a firm, the worker gets the job. If both persons apply to one firm, the firm hires each person at the probability $\frac{1}{2}$. An unemployed person gets a zero wage.

   (a) Let each worker has two available actions: applying to firm 1 or applying to firm 2. Construct the game matrix for this game, in which the payoff of each person is her expected hourly wage.

   (b) Find the two persons' best response functions.

   (c) Using your results in Part (b), find all the mixed-strategy Nash equilibria.

4. (30 points, 5 points each) Alice and Bob are playing "rock, scissor, paper". However, because Bob has only two fingers, he may only choose rock or scissor. Alice can choose all three options. As usual, when one wins, the winner gets payoff 1 and the loser gets payoff -1. When there is a tie, both players get payoff 0.

   (a) Construct the game matrix in which Alice is player 1 having three options and Bob is player 2 having two options.

   Hint. The payoff matrix should be three by two.

   (b) Identify one strictly dominated strategy to reduce the game matrix into a two by two one.

(c) Write down the best response functions of the two players.

Hint. You need to first formally define the two players' mixed strategies.

(d) Treat this game as a general game (i.e., ignore the fact that it is a zero-sum game) and find all the mixed-strategy Nash equilibria by finding the intersections of the best response functions.

(e) By utilizing the fact that this is a zero-sum game, formulate Alice's problem for finding her equilibrium mixed strategy as a linear program. Then solve the linear program by the graphical approach or simplex method.

Note. You may want to verify that your answer here is identical to that in Part (d).

(f) Similar to Part (e), formulate and solve Bob's linear program to find his equilibrium mixed strategy. Again, you should see that your answer is identical to that in Part (d).

(g) Using your results in Part (e) or (f), show that there is no pure-strategy Nash equilibrium for this game.

5. (20 points; 5 points each) In this problem, we will use three examples to illustrate the basic ideas of a new game format: dynamic games, in which players chooses their actions sequentially. In each of the following games, assume that player 1 chooses her action first, then player 2 observes player 1's action, and finally player 1 chooses his action.

(a) Consider the game "prisoners' dilemma"

	Denial	Confession
Denial	$-1, -1$	$-9, 0$
Confession	$0, -9$	$-6, -6$

In equilibrium, what will player 1 do? What will player 2 do? What will be their payoffs?

(b) Consider the game "Bach or Stravinsky"

	Bach	Stravinsky
Bach	$2, 1$	$0, 0$
Stravinsky	$0, 0$	$1, 2$

In equilibrium, what will player 1 do? What will player 2 do? What will be their payoffs?

(c) Consider the game "matching pennies"

	Head	Tail
Head	$1, -1$	$-1, 1$
Tail	$-1, 1$	$1, -1$

In equilibrium, what will player 1 do? What will player 2 do? What will be their payoffs?

(d) For each of these three games, do you prefer to be a leader (who acts first) or a follower (who acts later than the leader)? Is being a leader always beneficial?