Operations Research, Spring 2015
Homework 0

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1 The story

Mikasa sits in her office looking outside the window. After she retired from the army, she went to Hsinchu, Taiwan to work in a semiconductor manufacturing company, IEDO. As she is smart, strong, diligent, and able to pay full attention on what she is working on, she finished all the assigned jobs perfectly. These perfect records lead her to the current position, the head of the Operations Research department. She has been promoted to this position for two weeks.

Her eyes are still as beautiful as before. However, she is worrying about something. Multiple customers order multiple products, which may be produced at multiple factories. As the head of the OR team, Mikasa now is responsible for assigning manufacturing tasks to the multiple factories the company owns. She finds that the new job is really challenging, as there are many factors to consider. Moreover, the objective is somewhat defined ambiguously. The previous head, Eren, cannot manage this job in a good way and thus get transferred to another department. Mikasa is facing the same challenge now.

1.1 The company

IEDO, locating at Hsinchu, Taiwan, is a semiconductor manufacturer that produces chips and sells to downstream customers, who then assemble these chips into 3C products like

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computers, monitors, cell phones, air conditioners, etc. IEDO started its business almost 40 years ago. At this moment, it owns eight factories (fabrication sites, or fabs) and sells more than 8,000 kinds of products to around 400 domestic or overseas customers. It is one of the most influential semiconductor manufacturing companies in the world.

IEDO's products can be categorized into four classes: consumer, communication, computer, and industrial/standard. The communication can be further broken down to the segments as baseband, application processor, image processor, display drivers, etc. The industrial/standard class includes power management ICs, near field communications, audio codec, etc. These products may be made by different technology levels. IEDO currently runs various technology nodes, ranging from the old N90, N65, N40, N28, N20, to the most advanced N16.[1]

IEDO currently owns three 12-inch fabs, four 8-inch fabs, and one 6-inch fab. In a 12-inch fab, chips are made on wafers whose diameters are 12 inches. Therefore, the production process is generally more efficient in a 12-inch fab than in a 8-inch or 6-inch fab. Those technology nodes (N90, N65, ..., and N16) distribute in all these fabs.

1.2 The order allocation problem

Mikasa now needs to plan for IEDO's production processes for the next week. Her task is described as follows.[2]

Six customers, all big companies, place orders to IEDO. Each order contains a set of products and the associated order quantities. Each customer may place more than one orders. For confidentiality reasons, customers' names must be hided. Therefore, they will be called customers 1, 2, ..., and 6. IEDO currently produces twelve kinds of products, labeled as products 1, 2, ..., 12. The spreadsheet "Orders" records the current ten orders placed by the six customers. The numbers there are the order quantities of each item.

[1] Different numbers mean different semiconductor manufacturing processes. For example, N16 means the 16 nanometer technology node. The next generation, N10, is expected to be used starting in 2016. See, e.g., `http://en.wikipedia.org/wiki/10_nanometer` for more details.

[2] Though the problem is motivated by a real problem faced by a real company, it has been modified to fit the need of this course. In particular, the problem has been simplified by taking away a lot of factors to be considered in practice. Technical details that are domain-specific about semiconductor manufacturing are also removed. Though this makes the problem somewhat abstract, this allows one to work on it without having knowledge in semiconductor manufacturing. Nevertheless, please still try to see the connection between the problem presented here and the real-world business.

For example, order 1, placed by customer 1, requires 2000 units of item 4, 3000 units of item 5, etc. The sheet also records the price to collect upon completing that order. For example, if all the ordered items are delivered to customer 1, customer 1 will pay IEDO \$5,500.[3] As long as any single unit of an ordered item is not delivered, IEDO gets nothing.

IEDO owns eight fabs. As different fabs are constructed at different times, they have different capacities and different production rates. The spreadsheet "Fabs" records the current eight fabs, their capacities, and their production rates with respect to all items. For example, fab 1 in total has 840 machine hours as its capacity for the next week. To produce item 1 in fab 1, IEDO must spend 1 hour to produce 10 units. If that 1 hour is spent on producing item 2, IEDO will only get 5 units. Suppose fab 1 is asked to produce items 4 and 5 for order 1, it needs to allocate $\frac{2000}{10} + \frac{3000}{5} = 800$ hours on this task. There will be only $840 - 800 = 40$ hours remaining for other production tasks. Due to technology limits, fab 1 is unable to produce items 10 to 12.

Obviously, sometimes an order cannot be completed by one single fab. For example, order 1 cannot be completed by fab 1 because item 10, required in order 1, cannot be produced in fab 1. If an order is split and be completed by multiple fabs, there will be multiple deliveries to a customer. The shipping costs and compensation for bothering the customer together form order splitting costs. The spreadsheet "Splitting" records the splitting cost of each customer per splitting. For example, suppose order 1, which is placed by customer 1, is completed by three fabs, IEDO must pay $\$1000 \times 2 = \$2000$ as the order splitting cost. The profit of completing order 1 is thus only $\$5500 - \$2000 = \$3500$.

Mikasa's team is responsible for assigning orders to fabs and how to allocate the capacity of each fab to the assigned orders. While ensuring that each fab's assigned tasks do not exceed its capacity limitation, the objective is to earn as much profit (sales revenue minus splitting cost) as possible. As the head of the OR team, Mikasa knows that she must somehow guide her teammates to come up with a way to do the planning. As the same problem will occur again and again,[4] Mikasa really wants to have a systematic way for doing this.

[3] All these numbers have been normalized to keep the real information confidential.

[4] If we allow IEDO to keep inventories for future demands, the planning problems for all future weeks will become interdependent. To make our lives easier, let's assume that this is not allowed.

2 Your task

Please answer the following questions:

1. (20 points) Use whatever method you like, make a suggestion to Mikasa by telling her how to assign orders to fabs. Write down your assignments, indicate all job splitting, show that fab capacities are enough, and calculate your final profit. Try to earn as much profit as possible. The points you earn in this problem is $20(\frac{z}{z^*})$, where z is your profit and z^* is the maximum attainable profit. DO NOT describe your method; leave that to Problem 3.

2. (20 points) Let $I = \{1, ..., 12\}$ be the set of items, $J = \{1, ..., 10\}$ be the set of orders, and $K = \{1, ..., 8\}$ be the set of fabs. Let Q_{ij} be the order quantity (in units) of item $i \in I$ of order $j \in J$, R_{ik} be the production rate (in units per hour) of producing item $i \in I$ in fab $k \in K$, and C_k be the capacity (in hours) of fab $k \in K$.

   (a) (5 points) For each of Q_{ij}, R_{ik}, and C_k, determine whether it is a decision variable or a parameter.

   (b) (5 points) Find the values of Q_{18}, R_{52}, and C_3.

   (c) (10 points) Find $\max_{i \in I}\{Q_{ij}\}$, i.e., the order quantity of the item that is ordered the most in order j, for all $j \in J$.

3. (60 points) Describe the method you design for Mikasa's problem. 20 points will be based on how logical your method is. Please note that Mikasa's problem may become of a larger scale, i.e., with more customers, orders, and items, in the future. The values of parameters may also become different. Your method will also be graded for 20 points by considering its extendability. Finally, please explain your method in an easy-to-understand way. 20 points will be based on your presentation.

3 Submission rules

The deadline of this homework is 2pm, March 2, 2015. Please put a hard copy of the work into the instructor's mailbox on the first floor of the Management Building 2 by the due time. Works submitted between 2pm and 3pm will get 10 points deducted as a penalty. Submissions later than 3pm will not be accepted. Each student must submit her/his individual work.