Operations Research, Spring 2015
Homework 1

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1 Problems

1. (20 points; 10 points each) During the next six months IEDO company has following demands for air conditioners: month 1, 2500; month 2, 4000; month 3, 4500; month 4: 4200; month 5: 3800, month 6: 4400. Air conditioners can be produced in either Hsinchu or Taoyuan. It takes 2 hours of skilled labor to produce an air conditioner in Hsinchu, and 2.5 hours in Taoyuan. It costs \$400 to produce an air conditioner in Hsinchu, and \$350 in Taoyuan. During each month, each city has 4000 hours of skilled labor available. It costs \$80 to hold an air conditioner in inventory for a month. At the beginning of month 1, IEDO has 2000 air conditioners in stock.

   (a) Suppose IEDO must meet (on time) all the demands. Formulate an LP whose solution will tell IEDO how to minimize the cost of meeting air conditioner demands for the next six months.

   (b) Suppose those demands are the maximum number that IEDO may sell, and IEDO can decide the sales quantity in each month. Each air conditioner can be sold at \$600. Formulate an LP whose solution maximizes the profit of selling air conditioners for the next six months.

2. (10 points) n jobs must be assigned to m workers, where $n \leq 2m$. Each job requires exactly 1 hour no matter who works on it. The cost for worker i to 100% complete job j is C_{ij}, $i = 1, ..., m$, $j = 1, ..., n$. A job can be assigned to multiple workers. If that happens, costs are incurred at each worker proportional to the amount of job assigned to her/him. For example, suppose 20%, 30%, and 50% of job 1 is assigned to workers 4, 5, and 6, then the total cost for completing job 1 is $0.2C_{14} + 0.3C_{15} + 0.5C_{16}$. A worker can work for at most 2 hours. In other words, if one assigns 100% of job 2 and 60% of job 3 to worker 8, she/he will have only 0.4 hour left to work on other jobs. Formulate an LP that finds an assignment that completes all jobs with the minimum cost.

3. (20 points; 5 points each) Consider the following LP

$$\begin{aligned} \max \quad & x_1 + 2x_2 \\ \text{s.t.} \quad & x_1 - x_2 \leq 4 \\ & x_1 + x_2 \leq 9 \\ & x_1 \geq 0, x_2 \text{ urs.} \end{aligned}$$

   (a) Graphically solve the LP.

   (b) Find all the basic feasible solutions of the LP.

   (c) Use the simplex method with the smallest index rule to solve the LP.

   (d) Let constraint 2 become $2x_1 - x_2 \leq 10$. Use the simplex method to solve the new LP.

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4. (20 points; 10 points each) Consider the following LP:

$$\begin{aligned} \max \quad & x_1 + 2x_2 \\ \text{s.t.} \quad & x_1 - x_2 \leq 4 \\ & x_1 + x_2 + x_3 \leq 9 \\ & x_3 \geq 3 \\ & x_i \geq 0 \quad \forall i = 1, ..., 3. \end{aligned}$$

   (a) Find all its extreme points. DO NOT prove that they are extreme points; just list them.

   (b) Use the simplex method with the smallest index rule to solve it. Is there any iteration that has no improvement?

5. (20 points; 10 points each) Consider the following LP:

$$\begin{aligned} \max \quad & x_1 + 2x_2 \\ \text{s.t.} \quad & x_1 - x_2 \leq 4 \\ & x_1 + x_2 + x_3 \leq 7 \\ & x_3 \geq 3 \\ & x_i \geq 0 \quad \forall i = 1, ..., 3. \end{aligned}$$

   (a) Prove that it is degenerate.

   (b) Use the simplex method with the smallest index rule to solve it. Highlight those degenerate basic feasible solutions you meet during the search process. Is there any iteration that has no improvement?

6. (10 points; 5 points each) Consider the LP in Problem 4 (NOT that in Problem 5!). Let the slack variables for constraints 1, 2, and 3 be x_4, x_5, and x_6, respectively. Consider the basis $B = (x_1, x_3, x_5)$.

   (a) (0 point) Find A_B, A_N, c_B, c_N, and b.

   (b) (5 points) Use the reduced cost $\bar{c}_N^T = c_B^T A_B^{-1} A_N - c_N^T$ to find an entering variable. Let that entering variable be x_j.

   (c) (5 points) Use $A_B^{-1} b$ and $A_B^{-1} A_j$ to do the ratio test to find a leaving variable.

2 Submission rules

The deadline of this homework is 2pm, March 23, 2015. Please put a hard copy of the work into the instructor's mailbox on the first floor of the Management Building 2 by the due time. Works submitted between 2pm and 3pm will get 10 points deducted as a penalty. Submissions later than 3pm will not be accepted. Each student must submit her/his individual work.