Operations Research, Spring 2015
Homework 3 (Optional)

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May 30, 2015

1 Problems

1. (20 points; 5 points each) Consider the function $f(x) = ax^2 + bx + c$, where a, b, and c are fixed constants.

   (a) Find the gradient and Hessian of f.
   Hint. While gradients and Hessian matrices are introduced for multi-variate NLPs, they also apply to single-variate NLPs!

   (b) When is the Hessian positive semi-definite?

   (c) Under what condition is f concave over $\mathbb{R}$

   (d) Suppose the condition obtained in Part (c) is satisfied, find an optimal solution for

   $$\max_{x \in \mathbb{R}} f(x)$$

   as a function of a, b, and c.

2. (20 points; 5 points each) Consider an EPQ problem with annual demand D, unit holding cost h per year, unit ordering cost K, and production rate r units per year. Suppose we mistakenly believe that the production rate is $r' = \frac{r}{2}$ and the unit holding cost is $h' = 2h$.

   (a) What is the true EPQ q^*? What is the production quantity q' that we will choose based on our wrong belief?

   (b) What is $\frac{q'}{q^*}$?

   (c) When r approaches D, the EPQ formula tells us that q^* approaches infinity. Intuitively explain why.

   (d) When r approaches infinity, the EPQ formula tells us that q^* approaches the EOQ quantity. Intuitively explain why.

3. (20 points) A hot dog vendor sells hot dogs for \$25 each at a night market. It costs \$10 for making a hot dog. All the hot dogs he fails to sell by the midnight must be disposed at a unit disposal cost \$2. The daily demand is normally distributed with mean 250 and standard deviation 80.

   (a) What are the unit overage and underage costs?

   (b) What is the newsvendor quantity that minimize the expected overage and underage cost?

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(c) Suppose that minimum number of hot dogs made must be at least 270. Find an optimal quantity that minimizes the expected overage and underage cost.

(d) Find the newsvendor quantities when the unit disposal cost is \$0 and \$5. Do you see the newsvendor quantity goes up or down as the disposal fee goes up? Please briefly give an intuition reason for your observation.

4. (20 points; 5 points each) For each of the following functions, find the region over which the function is convex.

   (a) $f(x_1, x_2) = x_1 x_2$.

   (b) $f(x_1, x_2, x_3) = x_1^2 + \sqrt{x_2 + x_3}$ where the domain is $\{x \in \mathbb{R}^3 | x_1 + x_2 \geq 1\}$.

   (c) $f(x_1, x_2, x_3) = x_1^2 x_2 + 2x_2^2 x_3 + 3x_3^2$.

   (d) $f(x_1, x_2, ..., x_n) = (x_1 - a)^2 + \cdots + (x_n - a)^2$, where a is a given constant.

5. (20 points; 5 points) A seller is going to price an information good (e.g., a movie) in the next two periods. The prices in the two periods can be different. There are in total a consumers in the market. For each consumer, the reservation prices r_1 and r_2 for the good in these two days are uniformly distributed in $[0, u_1]$ and $[0, u_2]$, respectively. u_1 and u_2 are given constants satisfying $u_1 > u_2$. It is assumed that r_1 and r_2 are independent. A consumer will be willing to buy the good (e.g., buy a ticket to watch the movie) in period 1 if her/his r_1 is higher than p_1, the first-period price. A consumer will be willing to buy the good in period 2 if $r_2 > p_2$, where p_2 is the second-period price, and she/he does not buy the good in period 1. As this is an information good, there is no marginal cost. The licensing fee for the information good has been paid and thus a sunk cost.

   (a) Given p_1, find the expected number of consumers who will buy the good in period 1.

   (b) Given p_1 and p_2, find the expected number of consumers who will buy the good in period 2.

   (c) Formulate the seller's profit maximization problem as a nonlinear program.

   (d) Is the seller's problem convex?

2 Submission rules

This is an optional homework assignment. If you do not do it, your grades for Homework 0, 1, and 2 will together count for 10% of your semester grades (3.33% each). If you do it, each of Homework 0, 1, 2, and 3 will count for 2.5%.

If you choose to do it, the deadline of this homework is 2pm, June 8, 2015. Please put a hard copy of the work into the instructor's mailbox on the first floor of the Management Building 2 by the due time. Works submitted between 2pm and 3pm will get 10 points deducted as a penalty. Submissions later than 3pm will not be accepted. Each student must submit her/his individual work.