Operations Research, Spring 2015
Suggested Solution for Homework 3

Solution providers: Ling-Chieh Kung
Department of Information Management
National Taiwan University

1. (a) The gradient and Hessian are $[2ax + b]$ and $[2a]$, respectively.
   (b) When $a \geq 0$.
   (c) When $a \leq 0$.
   (d) An optimal solution is $x^* = -\frac{b}{2a}$.
2. (a) We have
$$q^* = \sqrt{\frac{2KD}{h(1-\frac{D}{r})}} \quad \text{and} \quad q' = \sqrt{\frac{2KD}{2h(1-\frac{D}{r/2})}}.$$
   (b) We have
$$\frac{q'}{q^*} = \sqrt{\frac{r-D}{r-2D}}.$$
   (c) When r approaches D, we need a very large lot size to satisfy all the demands.
   (d) When r approaches infinity, once we produce we get what we want in a very short time. This is very close to the case of EOQ, under which we get all we order at one single time point.
3. (a) The unit overage is \$12 and the unit underage cost is \$15.
   (b) 261.18 units.
   (c) 270 units.
   (d) The newsvendor quantities when the unit disposal cost is \$0 and \$5 are 270.27 and 250, respectively. The newsvendor quantity goes down as the disposal fee goes up because it becomes more costly to dispose unsold inventory, i.e., overage cost becomes higher.
4. (a) The function is nowhere convex.
   (b) The function is nowhere convex.
   (c) The function is convex if $x_2 \geq 0$ and $6x_2x_3 \geq 3x_1^2 + 4x_2^3$.
   (d) The function is convex everywhere.
5. (a) $a(1 - \frac{p_1}{u_1})$.
   (b) $a(\frac{p_1}{u_1})(1 - \frac{p_2}{u_2})$.
   (c) The seller's profit maximization problem is
$$\begin{aligned} \max_{p_1, p_2} \quad & a\left(1 - \frac{p_1}{u_1}\right)p_1 + a\left(\frac{p_1}{u_1}\right)\left(1 - \frac{p_2}{u_2}\right)p_2 \\ \text{s.t.} \quad & p_1 \geq 0, p_2 \geq 0. \end{aligned}$$
   (d) No, as the Hessian is not always positive semidefinite.