Operations Research, Spring 2015
Pre-lecture Problems for Lecture 6

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Note. The deadline of submitting the pre-lecture problem is ***9:10am, April 9, 2015***. Please submit a hard copy of your work in class. Late submissions will not be accepted. Each student must submit her/his individual work. Submit ONLY the problem that counts for grades.

1. (0 point) Find the dual for the following LP:

$$\begin{array}{rlllllll} \max & 4x_1 & - & 2x_2 & + & x_3 & & \\ \text{s.t.} & 2x_1 & + & x_2 & & & \leq & 10 \\ & & & x_2 & + & x_3 & \geq & 16 \\ & x_1 & & 3x_2 & - & 3x_3 & = & 14 \\ & \multicolumn{7}{l}{x_1 \geq 0,\ x_2 \leq 0,\ x_3 \text{ urs.}} \end{array}$$

2. (0 point) Consider a primal LP

$$\begin{aligned} \max \quad & 3x_1 + 5x_2 \\ \text{s.t.} \quad & x_1 + x_2 \leq 16 \\ & x_2 \leq 8 \\ & x_1 \geq 0, x_2 \geq 0. \end{aligned}$$

   (a) Find a primal optimal solution x^*.
   (b) Formulate the dual LP.
   (c) Solve the dual LP to get a dual optimal solution y^*. Show that $c^{\mathrm{T}}x^* = (y^*)^{\mathrm{T}}b$, where c and b are the primal and dual objective function.

3. (10 points) Consider the primal LP that you just solved in Problem 2.

   (a) Find a primal optimal basis B. Verify that $A_B^{-1}b = x_B^*$, the basic variables of the optimal solution x^* you found in Problem 2a.
   (b) Verify that $c_B^{\mathrm{T}}A_B^{-1} = y^*$, the dual optimal solution you found in Problem 2c.
   (c) Find the shadow prices for the two primal constraints.[1]

[1] If you are applying the correct concept, you may need no calculation for finding them. But maybe you would like to do a verification by calculating them by solving two modified primal LPs?