Operations Research, Spring 2015
Pre-lecture Problems for Lecture 12

Instructor: Ling-Chieh Kung
Department of Information Management
National Taiwan University

Note. You do not need to submit anything.

1. (0 point) For each of the following matrices, determine whether it is positive semi-definite.
   (a) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.
   (b) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$.
   (c) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 1 \\ 0 & 0 & 2 \end{bmatrix}$.
2. (0 point) For each of the following functions, find the region over which the function is convex.
   (a) $f(x) = x^3 + 2x^2 + x + 2$.
   (b) $f(x_1, x_2) = x_1^3 + 2x_2^2 + x_1 + 2$.
   (c) $f(x_1, x_2, x_3) = x_1^2 x_3 + 2x_2 x_3 + x_1 + 2$.
3. (0 point) Consider the following nonlinear program
$$\begin{aligned} \min \quad & (x_1 - 3)^2 + (x_2 - 2)^2 \\ \text{s.t.} \quad & 2x_1 + x_2 \leq 4. \end{aligned}$$
   (a) Prove or disprove that the NLP is a convex program.
   (b) Find the Lagrangian of this NLP. What is the sign of your Lagrangian multiplier?
   (c) Formulate the Lagrangian relaxation.
   (d) According to the FOC of the Lagrangian, find a necessary condition for any optimal solution.
   (e) Find an optimal solution for the NLP.